ANALISI MATEMATICA B
11.3.2022 - LEZIONE 65

ES

$$\int x \cdot e^{x^2-x} \cdot dx = \int \left[\frac{1}{2}(2x-1) e^{x^2-x} + \frac{1}{2} e^{x^2-x} \right] dx$$

$$= \frac{1}{2} e^{x^2-x} + \frac{1}{2} \int e^{x^2-x} dx$$

$$\int e^{t^2} dt \longleftrightarrow \int e^{-y^2} dy$$

$x^2-x = \left(x-\frac{1}{2}\right)^2 - \frac{1}{4}$

$e^{x^2-x} = e^{t^2-\frac{1}{4}} = \frac{e^{t^2}}{\sqrt[4]{e}}$

$t = \left(x-\frac{1}{2}\right)$

$t = iy$

$t^2 = -y^2$

$D e^{\lambda t} = \lambda \cdot e^{\lambda t}$ $\qquad \lambda \in \mathbb{C}$

$$\int \frac{P(x)}{Q(x)} dx \qquad P, Q \text{ polinomi.}$$

① $\deg P < \deg Q$

② Si fattorizza Q:

in $\mathbb{C}$: $Q(z) = c \cdot (z-\lambda_1) \cdots (z-\lambda_n)$

$n = \deg Q$

in $\mathbb{R}$:

Se $Q(x) \in \mathbb{R}[x]$ $\qquad$ Se $\lambda \in \mathbb{C}$, $Q(\lambda) = 0 \Rightarrow Q(\bar{\lambda}) = 0$

$$(z-\lambda)(z-\bar\lambda) = z^2 - (\lambda+\bar\lambda)z + \lambda\bar\lambda$$
$$= z^2 - 2\,\mathrm{Re}\lambda \cdot z + |\lambda|^2$$

Es

$$\int \frac{x+1}{x^2+x+1}\,dx$$

$x^2+x+1 \qquad \Delta = 1-4<0$

$D \operatorname{arctg} y = \frac{1}{1+y^2}$ (2)

$D \ln|x^2+x+1| = \frac{2x+1}{x^2+x+1}$ (1)

$$\int \frac{x+1}{x^2+x+1}\,dx = \frac{1}{2}\int \frac{2x+1+1}{x^2+x+1}\,dx$$

$$= \frac{1}{2}\ln|x^2+x+1| + \frac{1}{2}\int \frac{1}{x^2+x+1}\,dx$$

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4}\left(\left(\frac{x+\frac{1}{2}}{\frac{\sqrt3}{2}}\right)^2+1\right)$$

completamento del quadrato

$$\int \frac{1}{x^2+x+1}\,dx = \frac{4}{3}\int \frac{1}{\left(\frac{2x+1}{\sqrt3}\right)^2+1}\,dx = \dots \qquad \begin{cases} y = \frac{2x+1}{\sqrt3} \\ dx = \frac{\sqrt3}{2}\,dy \end{cases}$$

$$= \frac{4}{3}\cdot\frac{\sqrt3}{2}\cdot \operatorname{arctg}\frac{2x+1}{\sqrt3}$$

$$\int \frac{x+1}{x^2+x+1}\,dx = \ln\sqrt{x^2+x+1} + \frac{\sqrt3}{3}\operatorname{arctg}\frac{2x+1}{\sqrt3}.$$

Vediamo $\deg Q > 2$

$$\int \frac{x^3+1}{x^2(x^2+1)}\,dx \qquad \frac{A}{x}+\frac{B}{x^2}=\frac{Ax+B}{x^2}$$

$$\frac{x^3+1}{x^2(x^2+1)} = \frac{Ax+B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$= \frac{(Ax+B)(x^2+1)+(Cx+D)x^2}{x^2\cdot(x^2+1)}$$

$$= \frac{x^3[A+C]+x^2\cdot[B+D]+x\cdot A+B}{x^2\cdot(x^2+1)} \overset{!}{=} \frac{x^3+1}{x^2\cdot(x^2+1)}$$

$$\begin{cases} A+C=1 \\ B+D=0 \\ A=0 \\ B=1 \end{cases} \qquad \begin{cases} A=0 \\ B=1 \\ C=1 \\ D=-1 \end{cases}$$

$$\int \frac{x^3+1}{x^2\cdot(x^2+1)} = \int \frac{1}{x^2}\,dx + \int \frac{x-1}{x^2+1}\,dx = -\frac{1}{x}+\frac{1}{2}\ln(x^2+1) - \arctg x$$

$$\left[x^2+1+x^3-x^2 = x^3+1\right]$$

$$\frac{x-1}{x^2+1} = \frac{1}{2}\cdot\frac{2x}{x^2+1} - \frac{1}{x^2+1}$$

D

FRATTI SEMPLICI

$$(*) = \int \frac{x^{28}+1}{\boxed{x^3(x+1)^2(x-1)^4(x^2+1)^7(x^2+x+1)^3}}$$

$$\frac{Ax^2+Bx+C}{x^3} + \frac{Dx+E}{(x+1)^2} + \frac{Fx^3+Gx^2+Hx+I}{(x-1)^4} + \frac{Jx^{13}+\dots+W}{(x^2+1)^7} + \frac{Zx^5+\dots+\alpha}{(x^2+x+1)^3}$$

$$\int \frac{1}{(x^2+1)^2}\,dx = ?$$

decomposizione di Hermite.

$$(*) = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{x^2+x+1} +$$

$$+ \left[\frac{R(x)}{x^2\cdot(x+1)(x-1)^3(x^2+1)^6(x^2+x+1)^2} \right]'$$

$\deg R < 22$

29 incognite

$$\left[\%_{\circ} \right]' = \left[R(x)\cdot\frac{1}{x^2}\cdot\frac{1}{x+1}\cdot\frac{1}{(x-1)^3}\frac{1}{(x^2+1)^6} \right]'$$

$$= \frac{R'(x)}{x^2\cdot(x+1)\cdots(x^2+x+1)^2} - 2\frac{R(x)}{x^3\cdot(x+1)\cdots(x^2+x+1)^2}$$

$$- \frac{R(x)}{x^2(x+1)^2(x-1)^3\cdots} + -6\frac{R(x)(2x)}{x^2\cdots(x^2+1)^7} + \cdots$$

Es $\displaystyle\int \frac{1}{(x^2+1)^2}\,dx = ?$

$$\frac{1}{(x^2+1)^2} \overset{?}{=} \frac{Ax+B}{x^2+1} + \left[\frac{Cx+D}{x^2+1}\right]'$$

$$= \frac{Ax+B}{x^2+1} + \frac{C}{x^2+1} - \frac{(Cx+D)\cdot 2x}{(x^2+1)^2}$$

$1 = -(Ci+D)\cdot 2i$

$1 = 2C - 2iD$

$\begin{cases} C = 1/2 \\ D = 0 \end{cases}$

$$= \frac{(Ax+B)(x^2+1) + C(x^2+1) - (Cx+D)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{x^3\cdot A + x^2[B+C-2C] + x[A-2D] + B+C}{(x^2+1)^2}$$

$$\begin{cases} A = 0 \\ B - C = 0 \\ A - 2D = 0 \\ B + C = 1 \end{cases} \qquad \begin{cases} A = 0 \\ B = 1/2 \\ C = 1/2 \\ D = 0 \end{cases}$$

$$\frac{1}{(x^2+1)^2} = \frac{1/2}{x^2+1} + \left[\frac{\frac{1}{2}x}{x^2+1}\right]'$$

$$\int \frac{1}{(x^2+1)^2}\,dx = \frac{1}{2}\arctg(x^2+1) + \frac{1}{2}\frac{x}{x^2+1}$$

Verifica derivata:

$$\frac{1}{2}\,\frac{1}{1+x^2}+\frac{1}{2}\,\frac{(x^2+1)-x\cdot 2x}{(x^2+1)^2}$$

$$=\frac{1}{2}\,\frac{1}{1+x^2}+\frac{1}{2}\,\frac{1-x^2}{(x^2+1)^2}$$

$$=\frac{\frac{1}{2}(\cancel{x^2}+1)+\frac{1}{2}(1-\cancel{x^2})}{(x^2+1)^2}=\frac{1}{(x^2+1)^2}.$$