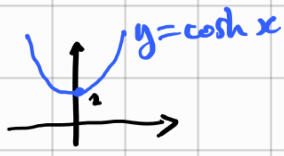
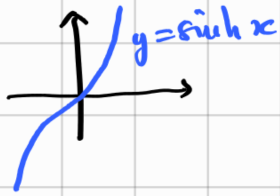
ANALISI MATEMATICA B
LEZIONE 62 - 4.3.2022

FUNZIONI IPERBOLICHE

$$\begin{cases} \cosh x = \dfrac{e^x + e^{-x}}{2} \\ \\ \sinh x = \dfrac{e^x - e^{-x}}{2} \end{cases}$$



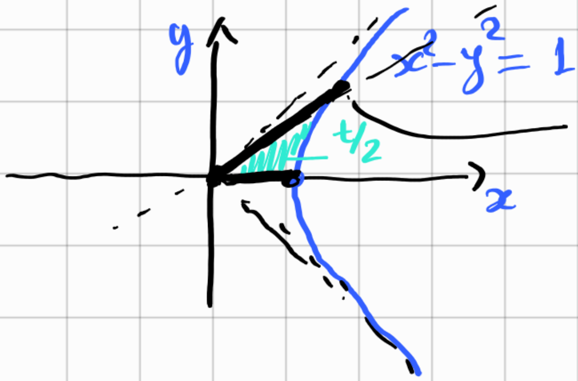


FUNZIONI CIRCOLARI

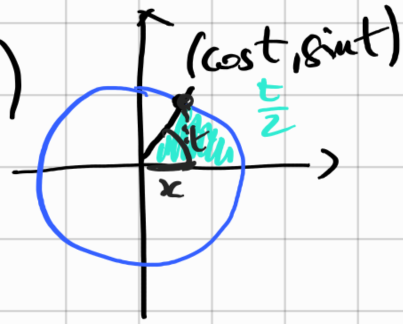
$$\begin{cases} \cos x = \dfrac{e^{ix} + e^{-ix}}{2} \\ \\ \sin x = \dfrac{e^{ix} - e^{-ix}}{2i} \end{cases}$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad\qquad \cos^2 x + \sin^2 x = 1$$

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{(\cancel{e^{2x}} + \cancel{e^{-2x}} + 2) - (\cancel{e^{2x}} + \cancel{e^{-2x}} - 2)}{4} = 1.$$



$(\cosh t, \sinh t)$



$$\begin{cases} \cosh' x = \sinh x \\ \sinh' x = \cosh x \end{cases} \qquad\qquad \begin{cases} \cos' = -\sin \\ \sin' = \cos \end{cases}$$

$$\begin{cases} \cosh(\alpha+\beta) = \cosh\alpha \cosh\beta + \sinh\alpha \sinh\beta \\ \\ \sinh(\alpha+\beta) = \sinh\alpha \cosh\beta + \cosh\alpha \sinh\beta \end{cases}$$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\beta\sin\alpha$

$$\cosh(\alpha+\beta) = \frac{e^{\alpha+\beta} + e^{-\alpha-\beta}}{2} =$$

$$\cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta = \frac{e^\alpha + e^{-\alpha}}{2} \cdot \frac{e^\beta + e^{-\beta}}{2} + \frac{e^\alpha - e^{-\alpha}}{2} \cdot \frac{e^\beta - e^{-\beta}}{2}$$

$$= \frac{e^{\alpha+\beta} + \cancel{e^{\alpha-\beta}} + \cancel{e^{\beta-\alpha}} + e^{-\alpha-\beta} + e^{\alpha+\beta} - \cancel{e^{\alpha-\beta}} - \cancel{e^{\beta-\alpha}} + e^{-\alpha-\beta}}{4}$$

$$= \frac{e^{\alpha+\beta} + e^{-\alpha-\beta}}{2} = \cosh(\alpha+\beta).$$

$\sinh : \mathbb{R} \to \mathbb{R}$ è bigettiva

$\text{settsinh} : \mathbb{R} \to \mathbb{R}$ è la funzione inversa.

$\cosh : [0,+\infty) \to [1,+\infty)$ è bigettiva

$\text{settcosh} : [1,+\infty) \to [0,+\infty)$ è la funzione inversa.

$$\text{settsinh}'(x) = \frac{1}{\cosh(\underbrace{\text{settsinh}\, x})} = \frac{1}{\sqrt{x^2+1}}$$

$$\cosh^2 - \sinh^2 = 1$$

$$\left[\begin{array}{l} \cosh x = \sqrt{1+\sinh^2 x} \\ \sinh x = \pm\sqrt{\cosh^2 x - 1} \end{array}\right]$$

$$\text{settcosh}'(x) = \frac{1}{\sinh(\text{settcosh}\, x)} = \frac{1}{\sqrt{x^2-1}}$$

$x \geq 1$

$\text{settsinh}\, x = y \geq 0$ t.c. $\quad \sinh y = x$

$$\frac{e^y - e^{-y}}{2} = x \qquad e^y - e^{-y} - 2x = 0$$

$$(e^y)^2 - 2x e^y - 1 = 0$$

$$e^y = x \pm \sqrt{x^2+1} \qquad y = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \ln\left(x + \sqrt{x^2+1}\right) = \text{settsenh}\, x$$

$\text{settcosh}\, x = y \geq 1 \qquad \cosh y = x$

$$\frac{e^y + e^{-y}}{2} = x \qquad (e^y)^2 - 2x e^y + 1 = 0$$

$$e^y = x \pm \sqrt{x^2-1} \overset{!}{\geq} 1 .$$

$$y = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\text{settcosh}\, x = \ln\left(x + \sqrt{x^2-1}\right) \in \int \frac{1}{\sqrt{x^2-1}}\,dx$$

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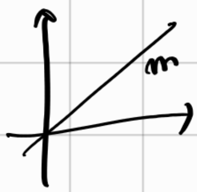
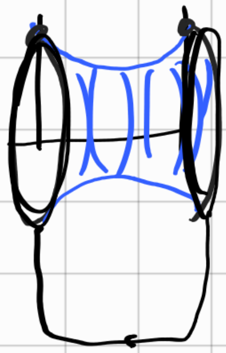
x^2+y^2 $\qquad$ $\underbrace{x^2-y^2}$ $\qquad$ $\underbrace{x^2+y^2+z^2-t^2}$



$$\text{tgh}\, x = \frac{\sinh x}{\cosh x}$$



RICERCA DELLA PRIMITIVA

Primitive che sono funzioni elementari

$$\frac{x^{\alpha+1}}{\alpha+1} \xrightarrow{D} x^\alpha$$

$$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1} \qquad \forall \alpha \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x|$$

$$\int e^x \, dx = e^x$$

$$\int \cos x \, dx = \sin x$$

$$\int \sin x \, dx = -\cos x$$

$$\int \frac{1}{1+x^2}\, dx = \text{arctg}\, x$$

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin x$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \text{settsinh}\, x$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \text{settcosh}\, x$$

LINEARITA'

$$\int (f+g) = \int f + \int g$$

$\lambda \in \mathbb{R}$

$$\int \lambda f = \lambda \int f$$

FORMULA PER IL CAMBIO DI VARIABILE

1. $$\int f(g(x))\, g'(x)\, dx = \left[\int f(y)\, dy\right]_{y=g(x)}$$

$$\begin{cases} y = g(x) \\ dy = g'(x)\, dx \end{cases}$$

$$y = g(x)$$

$$g'(x) = \frac{dy}{dx}$$

$$[F(y)]_{y=g(x)} = F(g(x))$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

Es $$\int (e^x)^2\, dx = \frac{1}{2}\int e^{2x}\, 2\, dx = \frac{1}{2}\int e^y\, dy$$

$$= \frac{1}{2}\int f(g(x))\, g'(x)\, dx \qquad \begin{cases} \boxed{y = 2x} \\ \underline{dy = 2\, dx} \end{cases}$$

$$\left[\begin{array}{l} f(y) = e^y \\ g(x) = 2x \end{array}\right] \qquad \| \qquad \frac{1}{2} e^y = \frac{1}{2} e^{2x}.$$

Es $$\int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2}\, dx = \frac{1}{2}\left[\int 1\, dx + \int \cos 2x\, dx\right]$$

$$\left[\begin{array}{l} \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 \\ \cos^2 x = \dfrac{1+\cos 2x}{2} \qquad \left[\cos x = \sqrt{\dfrac{1+\cos 2x}{2}}\right] \end{array}\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{2}\int \cos(2x)\, 2\, dx \right] = \frac{1}{2}x + \frac{1}{4}\int \cos y \, dy =$$

$y = 2x$

$dy = 2dx$

$$= \frac{1}{2}x + \frac{1}{4}\sin y = \frac{1}{2}x + \frac{1}{4}\sin 2x = \frac{x + \sin x \cdot \cos x}{2}$$

dim

$$\int \underbrace{f(g(x))\, g'(x)}\, dx = \left[\int f(y)\, dy \right]_{y = g(x)}.$$

$F \in \int f \qquad F' = f$

sul lato destro ho $\quad F(g(x))$.

Verifica: $\big(F(g(x))\big)' = F'(g(x)) \cdot g'(x)$

$$= f(g(x)) \cdot g'(x).$$