ANALISI MATEMATICA B
LEZIONE 46 - 24.1.2022

Nomenclatura

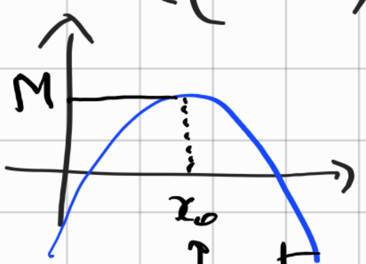
crescente / decrescente

(assoluto/globale) dominio

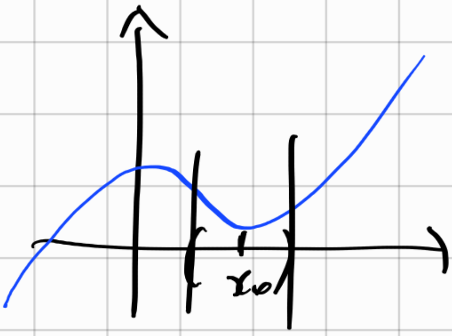
massimo/minimo < pto di max/min ; (valore) max, min

codominio

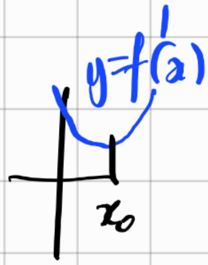
valore massimo → M



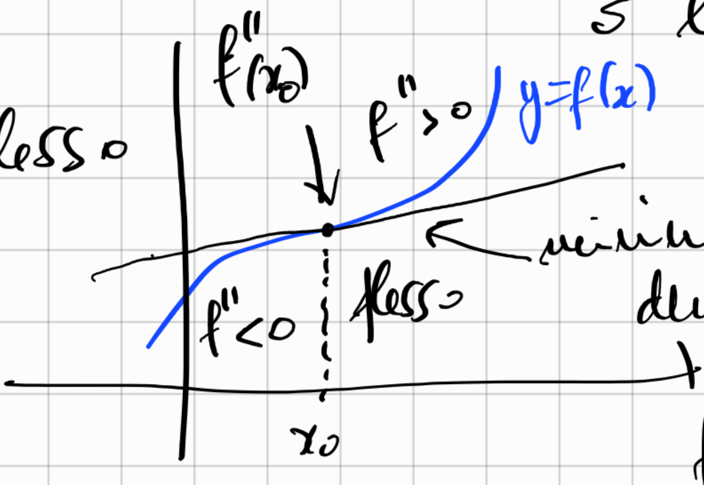
↑ pto di massimo



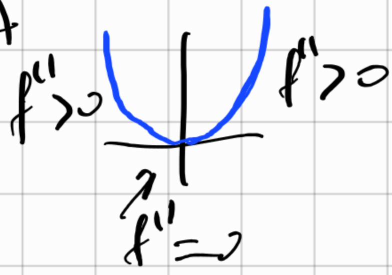
max/min relativo o locale

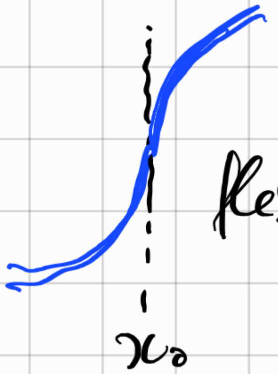


pti di flesso



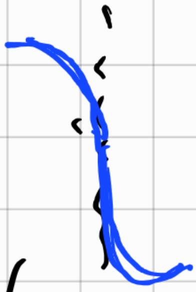
minimo della derivata





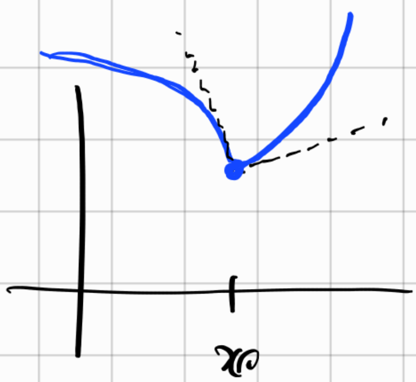
flesso verticale

derivata infinita.

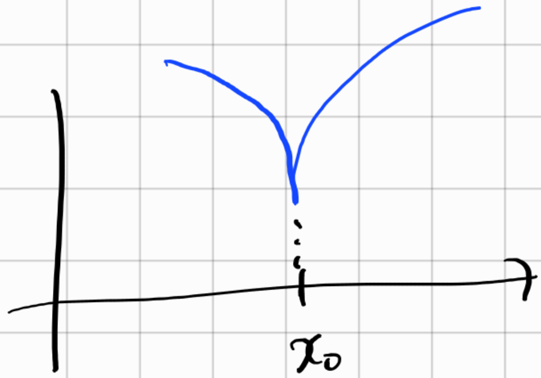


Es $y=\sqrt[3]{x}$

Punto angoloso



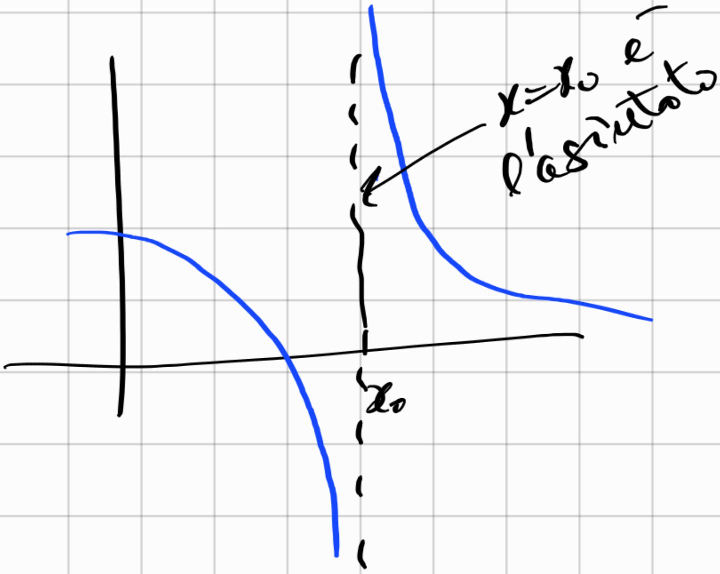
Punto di cuspide



Asintoto verticale

$$\lim_{x \to x_0^{\pm}} f(x) = \pm\infty$$

(discontinuità)



Asintoto orizzontale

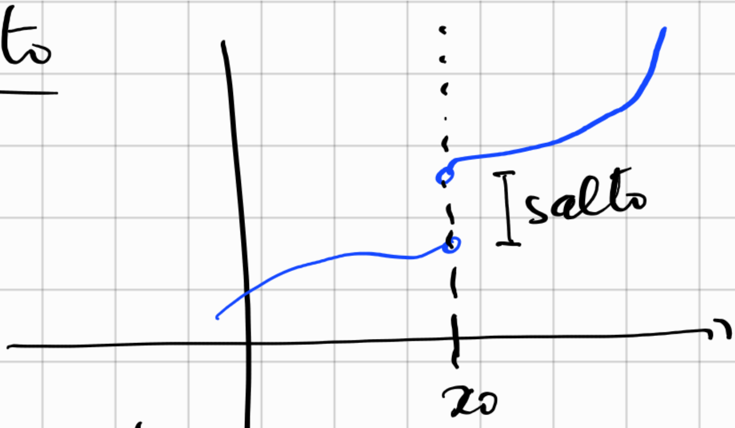
$$\lim_{x \to +\infty \, (-\infty)} f(x) = y_0 \in \mathbb{R}$$

Asintoto obliquo ($m \neq 0$)

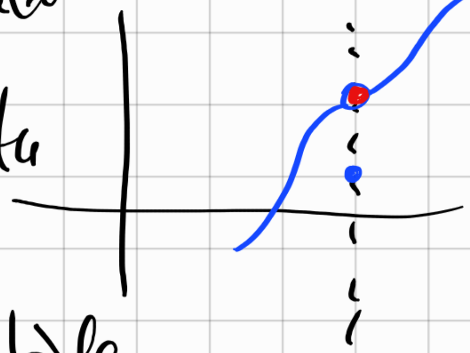
$$\lim_{x \to +\infty \, (-\infty)} f(x) - (mx + q) = 0$$

Discontinuità a salto

limiti destro/sin finiti ma diversi

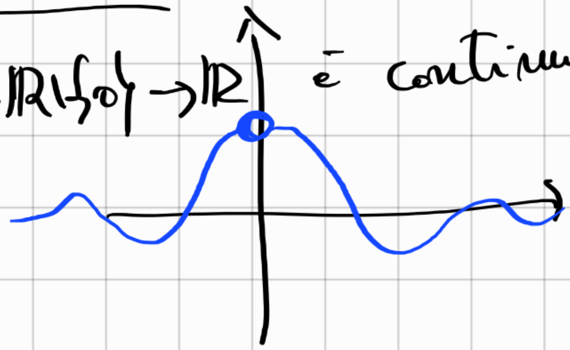


Il limite esiste ed è finito ma
$f(x_0)$ non è definita
o è diversa dal limite



Discontinuità eliminabile

ES $\quad f(x) = \dfrac{\sin x}{x} \quad f: \mathbb{R}\setminus\{0\} \to \mathbb{R}$ è continua.



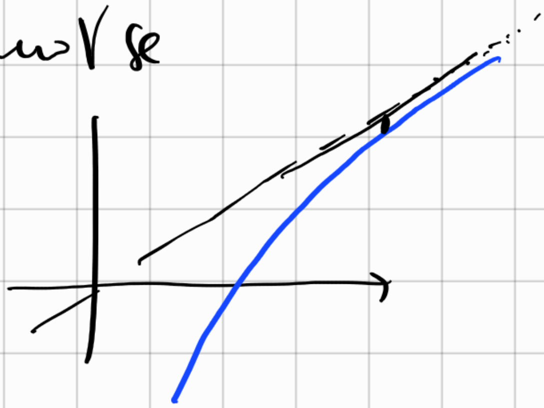
$$\tilde{f}(x) = \begin{cases} f(x) & \text{se } x \neq 0 \\ 1 & \text{se } x = 0 \end{cases}$$

$\tilde{f}$ è continua $\quad \tilde{f}: \mathbb{R} \to \mathbb{R}$

Come si trovano gli asintoti obliqui?

$y = mx + q$ è un asintoto obliquo per f se

$$\lim_{x \to +\infty} \underbrace{f(x) - (mx+q)} = 0$$



Se c'è un asintoto obliquo
lo trovo così

(1) $$\boxed{\lim_{x\to+\infty} \frac{f(x)}{x} = m}$$

per $x \to +\infty$ ($o -\infty$)

$$0 \leftarrow \frac{f(x)-(mx+q)}{x} = \boxed{\frac{f(x)}{x} - m} + \cancel{\frac{q}{x}}$$

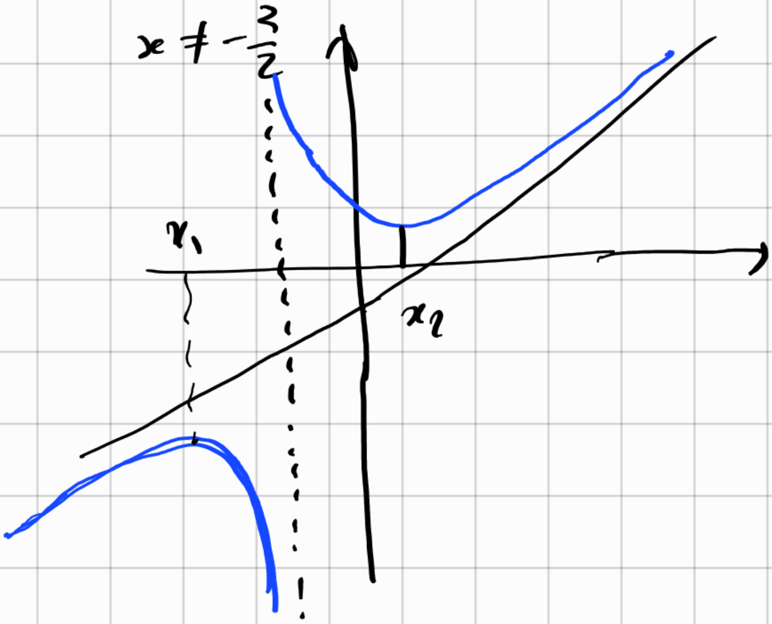
se ho l'asintoto obliquo

(2) $$\boxed{\lim_{x\to+\infty} f(x) - mx = q}$$

sse m e q esistono e sono finiti allora

$y = mx+q$ è un asintoto obliquo.

Es $$f(x) = \frac{x^2+1}{2x+3} \qquad x \neq -\frac{3}{2}$$



Asintoto obliquo:

$$\frac{f(x)}{x} = \frac{x^2+1}{2x^2+3x} \longrightarrow \frac{1}{2} = m \quad \text{per } x \to \pm\infty$$

$$f(x) - mx = \frac{x^2+1}{2x+3} - \frac{1}{2}x = \frac{\cancel{x^2}+1-\cancel{x^2}-\frac{3}{2}x}{2x+3} \longrightarrow -\frac{3}{4} = q$$

$y = \frac{x}{2} - \frac{3}{4}$ è asintoto obliquo

Studio il segno della derivata

$$f(x) = \frac{x^2+1}{2x+3}$$

$$f'(x) = \frac{2x(2x+3) - (x^2+1)\cdot 2}{(2x+3)^2}$$

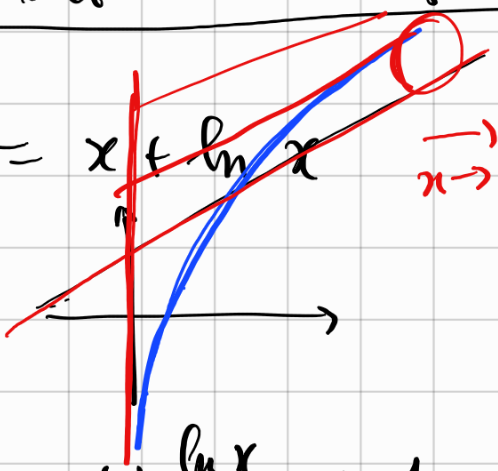
$$= \frac{4x^2+6x-2x^2-2}{(2x+3)^2} = 2\frac{x^2+3x-1}{(2x+3)^2}$$

$$x^2+3x-1=0 \qquad x_{12} = \frac{-3\pm\sqrt{9+4}}{2} = \frac{-3\pm\sqrt{13}}{2}$$

	x_1		$-\frac{3}{2}$		x_2	
f'	+ 0	−	$\nexists$	−	0	+
f	↗ max	↘		↘	min	↗

Es **Asintoto vs Asintotico**

$$f(x) = x + \ln x \qquad \xrightarrow[x\to+\infty]{} \quad \text{per } x \to +\infty \quad \boxed{f(x) \sim x}$$



$$\frac{f(x)}{x} = 1 + \frac{\ln x}{x} \longrightarrow 1.$$

$$\frac{f(x)}{x} = 1 + \frac{\ln x}{x} \longrightarrow 1 = m.$$

$$f(x) - mx = x + \ln x - x = \ln x \longrightarrow +\infty$$

per $x \to +\infty$.

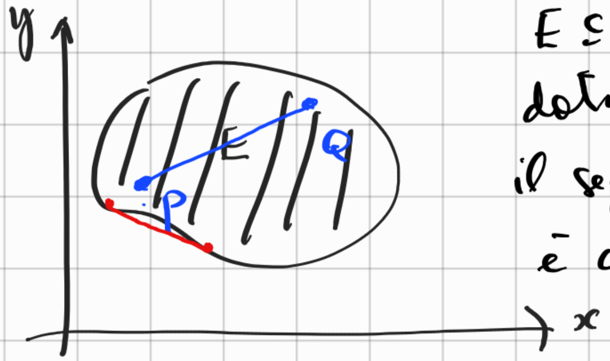
$f(x) \sim g(x)$ per $x \to x_0$ se $\frac{f(x)}{g(x)} \to 1$

vs

$f(x)$ ha asintoto $g(x)$ per $x \to x_0$

se $f(x) - g(x) \to 0$.

CONVESSITA'



$E \subseteq \mathbb{R}^2$ è convesso se dati comunque $P, Q \in E$ il segmento $[P,Q] = \overline{PQ}$ è contenuto in E:

$[P,Q] \subseteq E$.

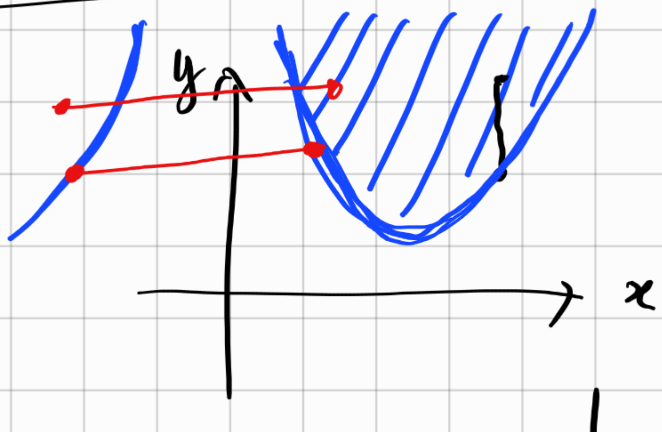
Dati $P, Q \in \mathbb{R}^2$

la retta per P e Q si parametrizza così:

$$t \mapsto (1-t)P + tQ$$

se $t \in [0,1]$ ho la parametrizzazione del segmento $[P,Q]$.

$f : A \subseteq \mathbb{R} \to \mathbb{R}$



diremo che f è convessa se

$$\text{epi}\, f := \{(x,y) : y \geq f(x),\ x \in A\}$$

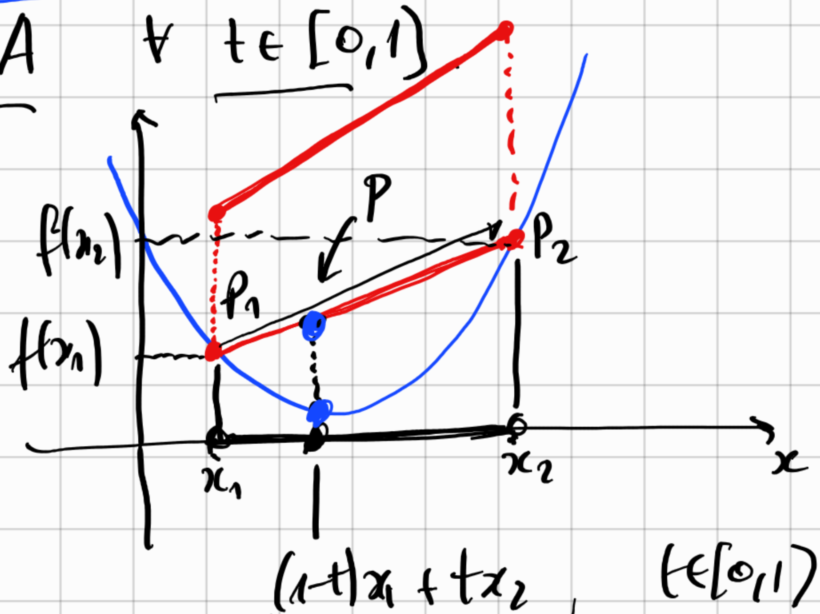
$f : A \subseteq \mathbb{R} \to \mathbb{R}$ è convessa se e solo se:

(i) A è un intervallo

(ii) $\boxed{f((1-t)x_1 + t x_2) \leq (1-t) f(x_1) + t f(x_2)}$

$\forall\ x_1, x_2 \in A \quad \forall\ t \in [0,1]$

$(x_1 < x_2)$

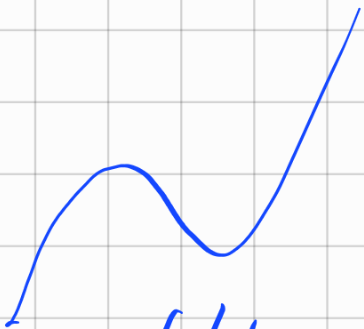


$$P = \Big(\underbrace{(1-t)x_1 + t x_2}_{x},\ \underbrace{\boxed{(1-t)f(x_1) + t f(x_2)}}_{y}\Big)$$

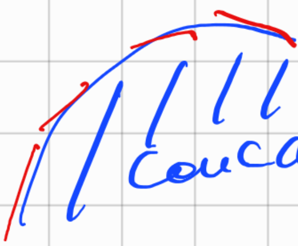
$P \in \operatorname{epi} f \iff y \geq f(x)$.

---

f è concava se $-f$ è convessa.



← non è né concava né convessa.

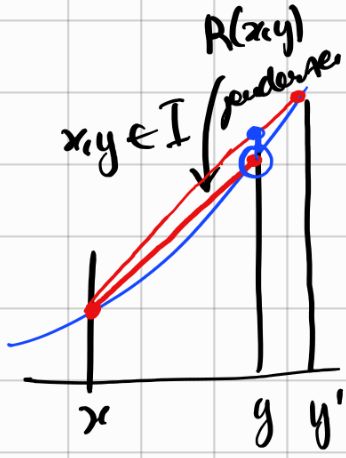


Lemma $f : I \to \mathbb{R}$, $I \subseteq \mathbb{R}$ intervallo

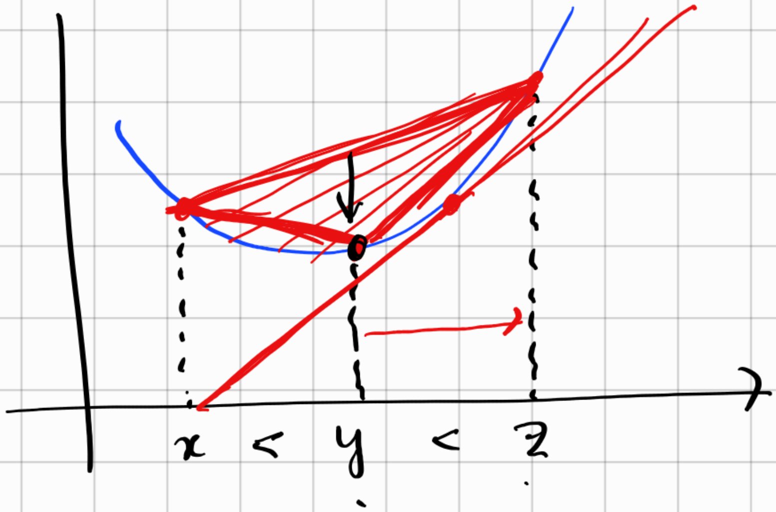
f è convessa se e solo se:

$\left[\text{posto } R(x,y) = \dfrac{f(y)-f(x)}{y-x} \right]$

|| $R(x,y)$ è crescente in entrambe le variabili.



$R(x,y') \geq R(x,y)$



$$\underbrace{R(x,y) \leq R(x,z)} \leq \underbrace{R(y,z)}$$