Future Perspectives on Linear and Nonlinear

Modelling of Contact-Type Perturbations

8-12 July 2024

TITLES and ABTRACTS

Speaker. *Badreddine Benhellal, Carl von Ossietzky Universität Oldenburg*

Title. *On Neumann-Poincaré operators and self-adjoint transmission problem*

Abstract. *In this talk, we discuss the self-adjointness in L^2-setting of the operators acting as $-\mathrm{div}\cdot h\nabla$, with piecewise constant functions h having a jump along a Lipschitz hypersurface Σ, without explicit assumptions on the sign of h. We establish a number of sufficient conditions for the selfadjointness of the operator with H^s-regularity for suitable $s \in [1, \frac{3}{2}]$, in terms of the jump value and the regularity and geometric properties of Σ. An important intermediate step is a link with Fredholm properties of the Neumann-Poincaré operator on Σ, which is new for the Lipschitz setting.*

Based on joint work with Konstantin Pankrashkin (Oldenburg).

Speaker. *Ivan Beschastnyi, INRIA Côte d'Azur University*

Title. *Normal operators of singular PDEs*

Abstract. *When studying differential operators it is quite common to reduce a given problem to the study of model operators, which capture their essential features. Normal operators constitute a geometric formalization of this notion. They are often used to construct exotic pseudo-differential calculi, but one can also use them directly to prove properties of operators via various shortcuts. In this talk I will explain what they are and how to use them to find closure of singular operators in various cases.*

Speaker. *Ugo Boscain, CNRS, Sorbonne Université, Paris*

Title. *Heat and Schrödinger evolution on surfaces embedded in 3D contact SR manifolds.*

Abstract. *In this talk I consider a surface embedded in a 3D contact sub-Riemannian manifold. Such a surface inherits a field of direction (with norm) from the ambient space. This field of directions is singular at characteristic points (i.e., where the surface is tangent to the set of admissible directions). In this talk we will study when the normed field of directions permits to give to the surface the structure of metric space (of "SNCF" type). I will also study how to define the heat and the Schrödinger equation on such a structure and if the singular points are "accessible" or not. When the singular points are accessible we will study self-adjoint extensions with Kirchhoff like boundary conditions.*

Speaker. *Luca Fanelli, University of the Basque Country, Bilbao*

Title. *Resolvent estimates for the Heisenberg Sublaplacian*

Abstract. *The sublaplacian in the Heisenberg Group is a canonical example of hypoelliptic operator, showing peculiar features along some natural time evolution flows. In particular, the associated Schrödinger evolution has a soliton solution, with a consequent lack of dispersion. In this seminar, we will show how to recover some dispersive estimates (Kato-Yakima local smoothing) through a suitable analysis of the resolvent operator, via Mourre's Theory. The results are obtained in collaboration with H. Mizutani (Osaka University), Luz Roncal and N. Schiavone (BCAM).*

Speaker. *Noriyoshi Fukaya, University of Waseda*

Title. *Uniqueness of ground states for 2d-nonlinear Schrödinger equations with point interaction*

Abstract. *We consider the nonlinear Schrödinger equations with point interaction in two dimensions. It is known that the ground states are positive, radial, and decreasing functions. In this talk, we show that the positive radial solutions of the corresponding equation are unique.*

Speaker. *Matteo Gallone, SISSA Trieste*

Title. *Ground States of NLS with Point Interaction and Coulomb Potential*

Abstract. *In this presentation, I will address the existence of ground states for a Nonlinear Schrödinger (NLS) equation in R^2 with a Coulomb potential and a point interaction. Initially, I will demonstrate how the Krein-Vishik-Birman extension theory can be effectively employed to construct the linear model. Following this, I will establish that, given a fixed mass and focusing on the L^2-subcritical power nonlinearity, a ground state indeed exists. Each of these ground states is positive, radially symmetric, and decreasing, exhibiting a logarithmic singularity at the origin. This work is based on a collaboration with F. Boni.*

Speaker. *Masahiro Ikeda, RIKEN Institute of Physical and Chemical Research*

Title. *On stability and instability of standing waves for 2D-nonlinear Schrödinger equations with point interaction*

Abstract. *This talk is based on a joint work with Professors Noriyoshi Fukaya and Vladimir Georgiev. We study existence and stability properties of ground-state standing waves for two-dimensional nonlinear Schrödinger equation with a point interaction and a focusing power nonlinearity. The Schrödinger operator with a point interaction describes a one-parameter family of self-adjoint realizations of the Laplacian with delta-like perturbation. The operator always has a unique simple negative eigenvalue . We prove that if the frequency of the standing wave is close to the negative eigenvalue, it is stable. Moreover, if the frequency is sufficiently large, we have the stability in the L^2-subcritical or critical case, while the instability in the L^2-supercritical case.*

Speaker. *Marilena Ligabó, University of Bari*

Title. *Truncated quantum observables and their semiclassical limit*

Abstract. *For quantum observables H truncated on the range of orthogonal projections Π_N of rank N ($[H,\Pi_N] \neq 0$), we study the corresponding Weyl symbol in the phase space in the semiclassical limit of vanishing Planck constant $\hbar \to 0$ and large quantum number $N \to \infty$, with $\hbar N$ fixed. Under certain assumptions, we prove the L^2- convergence of the Weyl symbols to a symbol truncated (hence, in general discontinuous) on the classically permitted region in phase space. As an illustration of the general theorems we analyse truncated observables for the harmonic oscillator and for a free particle in a one-dimensional box. In the latter case, we also compute the microscopic pointwise limit of the symbols near the boundary of the classically permitted region.*

Speaker. *Sandra Lucente, Dipartimento Interateneo di Fisica, Università degli Studi di Bari Aldo Moro*

Title. *Magnetic Nonlinear Klein-Gordon equation*

Abstract. *In this talk we consider Klein Gordon equation where standard derivatives are replaced by covariant derivatives those can represent the interaction with an electromagnetic potential. In a joint project with Luca Fanelli and Vladimir Georgiev, we obtain some crucial estimates for this equation, in particular conformal energy estimates, Morawetz type estimates and their applications.*

Speaker. *Diego Noja, University of Milan Bicocca*

Title. *On some asymptotic properties of NLS equation with a point interaction*

Abstract. *I will discuss some asymptotic properties of the solution of the time dependent NLS equation with a point interaction in dimension two and three. The first is the blow-up of the solution in dimension two and*

for supercritical nonlinearity, including the strong instability of standing waves. The second is the absence of scattering in both dimension two and three for low ("long-range") nonlinearities. If time permits I will also give perspectives on some further problems.

Speaker. *Tohru Ozawa, University of Waseda*

Title. *Proof of the Brezis-Gallouet inequality via heat semigroup*

Abstract. *Proof of the Brezis-Gallouet inequality is given by means of the heat semigroup in two space dimensions. This talk is based on my recent joint-work with Taiki Takeuchi, Kyoto University*

Speaker. *Eugenio Pozzoli, University of Rennes*

Title. *Small-time control of Schrödinger equations*

Abstract. *This talk is devoted to some recent results on the global approximate control, in arbitrarily small time, of bilinear Schrödinger equations. The main question in the following: given an initial state, which are the states that can be reached in approximately zero time, by suitably choosing arbitrarily large controls? Here, the spatial part of the control is fixed to be a potential $V(x)$, and the time-dependent part of the control $c(t)$ can be freely chosen, so that the control operator is $V(x,t) = c(t)V(x)$ and multiplies the state. A global answer to this question is still open. We will see that this problem can be nicely investigated with infinite-dimensional geometric techniques. In particular, for a Schrödinger equation on the torus, we will show how to build small-time controlled trajectories that follow arbitrary transport flows (that preserve the L^2-norm). We will then combine this fact with a celebrated result of Thurston on the simplicity of the group of diffeomorphisms, and obtain that compositions of the initial state with arbitrary diffeomorphisms of the torus are small-time approximately reachable states. This is a joint work with Karine Beauchard (ENS Rennes)*

Speaker. *Mario Rastrelli, University of Pisa and University of Waseda*

Title. *Perturbed Sobolev spaces in the L^r setting.*

Abstract. *We study the perturbed Sobolev space $H^{1,r}_\alpha(\mathbb{R}^2)$, $r \in (1,\infty)$, associated with singular perturbation Δ_α of Laplace operator in Euclidean space of dimension 2. The main results give the possibility to extend the L^2 theory of perturbed Sobolev space to the L^r case. When $r \in (1,\infty)$ we have appropriate representation of the functions in $H^{1,r}_\alpha$ in regular and singular part. An application to local well - posedness of the NLS associated with this singular perturbation.*

Speaker. *Raffaele Scandone, University of Napoli*

Title. *SOLITARY WAVES FOR NONLINEAR SCHRÖDINGER EQUATIONS WITH DELTA-LIKE INTERACTIONS*

Abstract. *I will consider a class of nonlinear Schrödinger equations in dimension $d = 2,3$. perturbed by a (suitably renormalized) delta potential. After introducing the Sobolev spaces adapted to the singular interaction, I will present some results on the existence and symmetry properties of solitary waves. Finally, I will discuss some connections with the topic of singular solutions to semilinear elliptic equations.*

Based on joint works with F. Boni, V. Georgiev, A. Michelangeli, D. Noja.

Speaker. *Nikolay Tzvetkov, École normale supérieure, Lyon*

Title. *Low regularity well-posedness of NLS on the two dimensional sphere*

Abstract. *We will consider the non linear Schrödinger equation, posed on the standard two dimensional sphere. We will explain how a randomisation of the initial data allows to go beyond the well-posedness result of Burq-Gérard and the speaker from 2003. This is a joint work with N. Burq, N. Camps and C. Sun.*

Speaker. *Luis Vega, University of the Basque Country and BCAM, Bilbao*

Title. *The binormal flow and the evolution of viscous vortex filaments.*

Abstract. *I'll present the so called Localized Induction Approximation that describes the dynamics of a vortex filament according to the Binormal Curvature Flow (BF). I'll give a result about the desingularization of the Biot-Savart integral proved with Marco A. Fontelos within the framework of Navier-Stokes equations. Some particular examples regarding BF obtained with Valeria Banica will be also considered. These examples allow to connect BF with the so-called Riemann non-differentiable function and the Frisch-Parisi approach to turbulence.*

Speaker. *Nicola Visciglia, University of Pisa*

Title. *Global existence for the generalized derivative NLS*

Abstract. *We prove that H^2 local solutions to the generalized derivative NLS can be extended globally in time, provided that we assume an a-priori H^1 uniform bound on the solution. As a consequence, due to the Hamiltonian structure of the equation, we get global existence for H^2 solutions under a smallness assumption in H^1. This is a joint work with M. Hayashi and T. Ozawa.*

Speaker. *Ivana Vojnović, University of Novi Sad, University of Wroclaw*

Title. *Generalised solutions to differential equations with point defect*

Abstract. *In this talk I will give an overview of approaches for solving nonlinear differential equations based on generalized algebras. In such constructions especially important are properties of associativity and compatibility of solutions, and we will see examples that illustrate meaning of these notions. Our preferred approach is based on the Colombeau-type algebras, basic constructions and properties of which will be explained.*

As an application, we will consider Schrödinger equations (and their approximations) of Hartree and cubic type in $\mathbb{R}^3$ with singular, point-like

perturbations (delta potential). We will study generalized solutions in the appropriate Colombeau-type algebra and explore the compatibility between Colombeau solutions and strong solutions.

The talk is based on joint works with Nevena Dugandžija and Alessandro Michelangeli.