Analisi Matematica 2 - Programma del corso - English version
Università degli Studi di Napoli Federico II
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Sito del corso: `http://www.velichkov.it/analisi2.html`

Corsi di Laurea:

- N36 Ingegneria Biomedica
- N39 Ingegneria dell'Automazione
- N43 Ingegneria Elettronica
- N46 Ingegneria Informatica
- P39 Ingegneria delle telecomunicazioni e delle media digitali

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Topology in $\mathbb{R}^n$

The space $\mathbb{R}^n$. Scalar product, Cauchy-Schwartz inequality, Euclidean distance, triangular inequality convergence of sequences in $\mathbb{R}^n$, Density of $\mathbb{Q}^n$ in $\mathbb{R}^n$.

Open and closed sets. Open sets. Union and intersection of closed sets. Product of open sets. Closed sets. Union and intersection of closed sets. Product of closed sets. Closed sets - definition by sequences. Closure, interior and boundary of a set in $\mathbb{R}^n$.

Compact sets. Coverings - open, finite and countable coverings. Compact sets - definition by open coverings, definition by converging sequences, definition as closed and bounded sets. Equivalence of the three definitions.

Continuous functions. Induced topology on subsests of $\mathbb{R}^n$. Continuous functions. Defition with open sets and definition with converging sequences. Composition of continuous functions. Continuous functions on compact sets. Theorem of Weierstrass.

Connected sets. Connected sets and sets connected by arcs. An open set is connected if and only if it is connected by arcs. Connected sets on the real line. Continuous functions and connected sets - the image of a connected set remains connected.

Partial derivatives

Partial derivatives and differentiability. Partial derivatives. Functions differentiable at a point. The differentiable functions are continuous. The differentiable functions admit partial derivatives in all directions. Example of a functions that admits partial derivatives but is not continuous. Example of a continuous function that has partial derivatives in all directions, but is not differentiable in zero. C^1 and C^2 functions. Differentiability theorem. Higher order partial derivatives. Lemma di Schwarz. Composition of differentiable functions.Hessian matrix. Second order Taylor expansion.

Local maxima and minima. Interior local maxima and minima. First order necessary condition. Critical points. Second order necessary condition. Second order sufficient conditions. Positively and negartively definite matrices. Local maxima and minima on the boundary of a regular set. Necessary and sufficient conditionsat first and second order in dimension two. Normal and tangent vectors to the boundary of a regular set in dimension two.

Implicit function theorem. Impicit function theorem in dimension two. Lagrange multiplyers' theorem and applications.

Differential forms and integration over curves

Differential forms in $\mathbb{R}^n$. 1-forms, 2-forms and k-forms in $\mathbb{R}^n$ - definitions. Sum of wo differential forms (of the same order). The product of a differential form and a function. Exterior product of differential forms. The differential of a function. Exterior derivative. Closed forms and exact forms. The exact forms are closed. Example of a closed form which is not exact.

Integration of 1-forms over curves. Curves in $\mathbb{R}^n$. C^1 regular curves, piecewise C^1 regular curves, simple curves, and closed curves. Equivalent curves. Opposite curves. Concatenation of curves. Integration of a 1-form over a piecewise C^1 regular curve. Integration over equivalent curves. Integration over opposite curves. Integration and concatenation of curves. Characterization of the exact 1-forms by integration over closed curves. In a rectangular domain the closed 1-forms are exact. In a star-shaped domain the closed 1-forms are exact. Derivation under the sign of the integral. Diffeomorphisms. In $\mathbb{R}^2$ the ball B_1 and the annulus $B_1 \setminus \{0\}$ are not diffeomorphic. In $\mathbb{R}^3$ the thorus and the ball are not diffeomorphic.

Integration of functions over curves. Integral of a continuous function over a piecewise C^1 curve. Integration over equivalent curves. Integration over opposite curves. Integration over concatenated curves. Approximation of the integral over a curve. Length of a curve.

Integration in $\mathbb{R}^n$.

Riemann integration in $\mathbb{R}^n$. Integration in rectangular domains. Partitions of a rectangular domain.Riemann sums. Definition of Riemann integrable function over a rectangular domain. Fubini's Theorem on rectangular domains. Integrability of continuous functions on rectangular domains. The integral of a bounded function over a bounded set. Integrability of continuous functions over normal domains (domains between the graphs of two functions). Fubini's Theorem over normal domains.

Integration by parts and the Divergence Theorem. Gauss-Green formulas in two-dimensional normal domains. The diveregence theorem over normal domains in dimension two. Partitions of unity in two-dimension regular domains. Divergence theorem in two-dimensional regular domains. Applications of the divergence theorem - Laplacian and Heat equation in dimension two.

Stokes' formula. Stokes' Theorem over normal domains in dimension two. Orientation in dimension two. Curves that parametrize clouner-clock-wisely the boundary of a regular set. Stokes' formula over regular domains in $\mathbb{R}^2$. Gauss-Green formulas as a consequence from the Stokes' Theorem. Diffeomorphisms and orientation in dimension two. Change of variables in two-diemsnional integrals. Integration in polar coordiantes. The integral of the Gaussian function.

Integration over parametric surfaces. Parametric surfaces in dimension two. Equivalent surfaces. Integration of two-forms over parametric surfaces. Integration over equivalent surfaces. Stokes' formula for surfaces. Vector product in $\mathbb{R}^3$. Divergence and curl of a vector field in $\mathbb{R}^3$. Normal vector to a parametric surface. Integral of functions over parametric surfaces. Curl Theorem. Application of the curl theorem - Faraday's law and the third Maxwell's equation.