

## ALGEBRA IN THE SPACE OF ULTRAFILTERS AND RAMSEY THEORY

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More than 35 years ago I gained some fame by producing a difficult and complicated proof of an easy theorem. Of course, the theorem was not known to be easy at the time. This was because there was no one who was simultaneously aware of all of the following facts, even though each fact was known by a large number of people:

- (1) The result was simply stated as a question about ultrafilters on  $\mathbb{N}$ .
- (2) There is an operation  $+$  on the Stone-Čech compactification  $\beta\mathbb{N}$  of  $\mathbb{N}$  which extends addition on  $\mathbb{N}$  and makes  $(\beta\mathbb{N}, +)$  a compact right topological semigroup.
- (3) Any compact Hausdorff right topological semigroup has idempotents.
- (4) The Stone-Čech compactification  $\beta\mathbb{N}$  of  $\mathbb{N}$  can be naturally viewed as the set of all ultrafilters on  $\mathbb{N}$ .

I never understood the original complicated proof (no, I did not plagiarize it), so when I was made aware of the above facts, I began a career-long love affair with the algebraic structure of the set of ultrafilters on a discrete semigroup  $S$ . I shall attempt to describe some of the high points of this affair. (Unfortunately, for those familiar with the US movie rating system, they are all rated G.)

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