

Principali formule di Taylor di punto iniziale $x_0 = 0$

$$e^x = \sum_{k=0}^n \frac{x^k}{k!} + o(x^n) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$\begin{aligned} \operatorname{sen} x &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\ &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \end{aligned}$$

$$\begin{aligned} (1+x)^\alpha &= \sum_{k=0}^n \binom{\alpha}{k} x^k + o(x^n) \\ &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots + \\ &\quad + \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-(n-1))}{n!} x^n + o(x^n) \end{aligned}$$

In generale :

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\dots(\alpha-(k-1))}{k!} \quad \text{se } k \neq 0$$

$$\binom{\alpha}{0} = 1$$

$$\frac{1}{1+x} = \sum_{k=0}^n (-1)^k x^k + o(x^n) = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + o(x^n).$$

$$\frac{1}{1-x} = \sum_{k=0}^n x^k + o(x^n) = 1 + x + x^2 + x^3 + \dots + x^n + o(x^n).$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \sum_{k=3}^n (-1)^{k-1} \frac{1 \cdot 3 \cdot 5 \dots (2k-3)}{2 \cdot 4 \cdot 6 \dots (2k)} x^k + o(x^n)$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3}{8}x^2 + \sum_{k=3}^n (-1)^k \frac{1 \cdot 3 \cdot 5 \dots (2k-1)}{2 \cdot 4 \cdot 6 \dots (2k)} x^k + o(x^n)$$

$$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{x^2}{2} + \frac{3}{8}x^4 + \sum_{k=3}^n \frac{1 \cdot 3 \cdot 5 \dots (2k-1)}{2 \cdot 4 \cdot 6 \dots (2k)} x^{2k} + o(x^{2n+1})$$

$$\frac{1}{1+x^2} = \sum_{k=0}^n (-1)^k x^{2k} + o(x^{2n+1})$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^n}{n!} + o(x^n)$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots + (-1)^n \frac{x^{2n}}{n!} + o(x^{2n+1})$$

$$a^x = e^{x \log a} = 1 + x \log a + \frac{x^2}{2!} \log^2 a + \dots + \frac{x^n}{n!} \log^n a + o(x^n)$$

$$\cos x = \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\log(1+x) = \sum_{k=1}^n (-1)^{k+1} \frac{x^k}{k} + o(x^n)$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1} \frac{x^n}{n} + o(x^n)$$

$$\log_a(1+x) = \frac{\log(1+x)}{\log a} = \frac{1}{\log a} \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1} \frac{x^n}{n} \right) + o(x^n)$$

$$\operatorname{arctg} x = x - x^3/3 + x^5/5 + \dots$$

$$\operatorname{arcsen} x = x + x^3/6 + 3x^5/40 + \dots$$

$$\operatorname{arccos} x = \pi/2 - x - x^3/6 - 3x^5/40 + \dots$$

$$\operatorname{tg} x = x + x^3/3 + 2x^5/15 + o(x^5).$$