Istituzioni di Matematiche

Prova del 29.06.09

1. CE $\begin{cases} |x|/\sqrt{2x^2-2x+1} \leq 1 \\ 2x^2-2x+1>0 \end{cases} \iff \forall x \in \mathbb{R}$

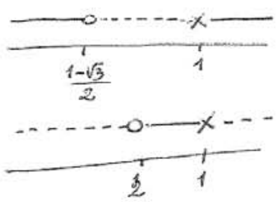
LIM per $x \to +\infty$ $\quad f(x) \to +\infty \quad y = x + \frac{\pi}{4}$

per $x \to -\infty$ $\quad f(x) \to -\infty \quad y = x + \frac{3}{4}\pi$

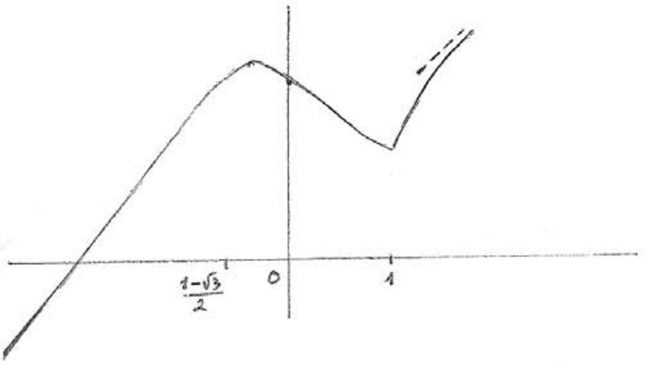
$f(0) = \frac{\pi}{2}$.

DRV $\quad f'(x) = 1 + \dfrac{2\,\mathrm{sgn}(x-1)}{2x^2-2x+1}$

DRV^2 $\quad f''(x) = \dfrac{-4(2x-1)\,\mathrm{sgn}(x-1)}{2x^2-2x+1}$



$f'(1^+) = 3$
$f'(1^-) = -1$ pto. angoloso



2. $-x^2+x+2 = \frac{9}{4} - (x-\frac{1}{2})^2$.

Si pone $x - \frac{1}{2} = \frac{3}{2}\,\mathrm{sen}\,t \quad (t \in [-\frac{\pi}{2}, \frac{\pi}{2}])$

$$\int_{-\pi/2}^{\pi/2} \frac{1+3\,\mathrm{sen}\,t}{2}\,dt = \frac{\pi}{2}.$$

4. $a_n \sim \dfrac{2\lg n}{n^{3/2}} < \dfrac{2n^{\alpha}}{n^{3/2}} = \dfrac{2}{n^{3/2-\alpha}}$

scegliamo α t.c. $\frac{3}{2} - \alpha > 1$, cioè $0 < \alpha < \frac{1}{2}$.

Il procedimento permette di concludere che la serie data è CONVERGENTE.

CE. $\quad y \geq 1, \quad x \in \mathbb{R}$

$y = 1$ soluzione costante

Altre soluz:

$$\int_{y_0}^{y} \frac{ds}{s\sqrt{s-1}} = \int_{x_0}^{x} s\,ds$$

$\Downarrow$

$\sqrt{s-1} = t$

$$\int_{\sqrt{y_0-1}}^{\sqrt{y-1}} \frac{2}{t^2+1}\,dt = \frac{x^2 - x_0^2}{2}$$

$2\,\mathrm{arctg}\sqrt{y-1} = \dfrac{x^2-c}{2}$

$\mathrm{arctg}\sqrt{y-1} = \dfrac{x^2-c}{4}$

$\sqrt{y-1} = \mathrm{tg}\,\dfrac{x^2-c}{4} \quad$ con $-\frac{\pi}{2} < \frac{x^2-c}{4} < \frac{\pi}{2}$

$y = 1 + \mathrm{tg}^2\,\dfrac{x^2-c}{4} \quad$ con $0 < \frac{x^2-c}{4} < \frac{\pi}{2}$

$\downarrow$

$c < x^2 < 2\pi + c$

se $c > 0 \quad \sqrt{c} < |x| < \sqrt{2\pi+c}$

se $-2\pi < c < 0 \quad |x| < \sqrt{2\pi+c}$