1.

C.E. $\quad \mathbb{R}$

SGN $\quad f(x)>0$ per $x \in (-1,0) \cup (0,+\infty)$

$f(x)<0$ per $x \in (-\infty,-1)$

$f(x)=0$ per $x=0$ e $x=-1$

LIM $\quad x \to \pm\infty \quad f(x) \sim x \lg|x| \to \pm\infty$ (senza asintoto)

DRV $\quad f'(x) = \lg(1+2|x|) + 2\,\mathrm{sgn}\,x \dfrac{1+x}{1+2|x|} \qquad x \neq 0$

$$f''(x) = \frac{4(\mathrm{sgn}\,x + x - 1)}{(1+2|x|)^2}$$

SGN f''

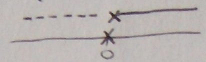
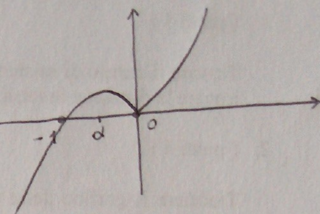
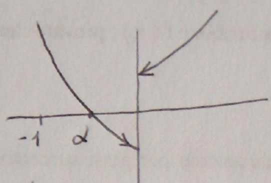


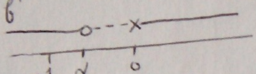
Grafico di f



f'



SGN f'



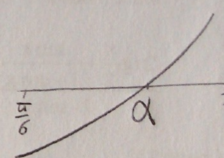
$x=0$ punto angoloso

2.

$f(x) = x - \mathrm{sen}\,2x + 1/4 \quad , \quad x \in [\pi/6, \pi/4]$

$f'(x) = 1 - 2\cos 2x > 0$

$f''(x) = 4\,\mathrm{sen}\,2x > 0$



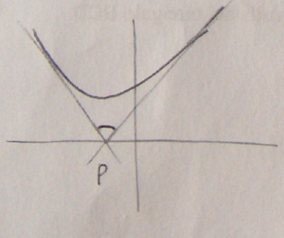
$$x_0 = \pi/4$$

$$x_{n+1} = x_n - \frac{x_n - \mathrm{sen}\,2x_n + 1/4}{1 - 2\cos 2x_n}$$

$$x_1 = \frac{3}{4} = 0{,}75$$

$$x_2 \cong 0{,}747$$

3.



$P = (a, 0)$

$$\begin{cases} y = m(x-a) \\ y = x^2+x+1 \end{cases} \Rightarrow \quad x^2 + (1-m)x + 1 + ma = 0$$

$\Delta = m^2 - 2(1+2a)m - 3$

$\Delta = 0$ per $m = 1 + 2a \pm 2\sqrt{a^2+a+1}$

$\operatorname{tg}\alpha_1 = 1 + 2a + 2\sqrt{a^2+a+1}$ $\qquad$ $\operatorname{tg}\alpha_2 = 1 + 2a - 2\sqrt{a^2+a+1}$

$$\operatorname{tg}\alpha = \operatorname{tg}(\alpha_2 - \alpha_1) = \frac{\operatorname{tg}\alpha_2 - \operatorname{tg}\alpha_1}{1 + \operatorname{tg}\alpha_2 \operatorname{tg}\alpha_1} = 2\sqrt{a^2+a+1}$$

$\operatorname{tg}\alpha$ minima per $a = -\frac{1}{2}$

4.

Condizione di continuità in $x = 1$ $\qquad$ $b + 5 = a + 2$

Condizione di derivabilità in $x = 1$ $\qquad$ $\begin{cases} b+5 = a+2 \\ 2b+5 = a \end{cases} \iff \begin{matrix} a = 1 \\ b = -2 \end{matrix}$

$f(2) = 4, \ f(0) = 0$

$\dfrac{f(2) - f(0)}{2 - 0} = f'(\xi) \iff$

In $[0, 1]$ deve essere

$2 = -4\xi + 5 \Rightarrow \xi = 3/4$

In $[1, 2]$ deve essere

$2 = 1$ assurdo

5.

$\operatorname{arctg} x \sim x - \frac{x^3}{3}$

$\lg(1 + x^2) \sim x^2 - \frac{x^4}{2}$

$\lg(1 + x \operatorname{arctg} x) \sim \lg(1 + x^2 - \frac{x^4}{3}) \sim (x^2 - \frac{x^4}{3}) - \frac{1}{2}x^4 = x^2 - \frac{5}{6}x^4$

Numeratore $\sim x^4/3$

$1 - \cos x \sim \frac{1}{2}x^2 - \frac{1}{24}x^4$

$\lg\cos x \sim \lg(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4) \sim (-\frac{1}{2}x^2 + \frac{1}{24}x^4) - \frac{1}{2}(\frac{1}{4}x^4) = -\frac{1}{2}x^2 - \frac{1}{12}x^4$

Denominatore $\sim -x^4/8$

Limite $= -8/3$.