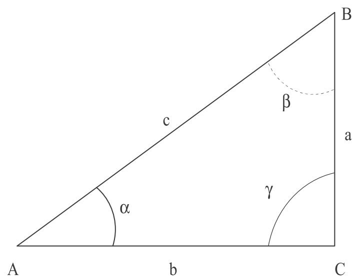
CAPITOLO VIII - INTRODUZIONE ALLA TRIGONOMETRIA

Relazioni tra gli elementi di un triangolo rettangolo

$\sin\alpha = \frac{BC}{AB}$ $\qquad$ $\cos\alpha = \frac{AC}{AB}$

$a = c\sin\alpha$ $\qquad$ $b = c\cos\alpha.$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}. \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha}.$$



Relazione tra gradi e radianti

$$\alpha^o : \alpha_{rad} = 180^o : \pi$$

Relazione Pitagorica o identità fondamentale della trigonometria e formule derivate

$$\boxed{(\sin\alpha)^2 + (\cos\alpha)^2 \quad = \quad 1}$$

Noto	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
$\sin\alpha$	$\sin\alpha$	$\pm\sqrt{1-\sin^2\alpha}$	$\dfrac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}$	$\dfrac{\pm\sqrt{1-\sin^2\alpha}}{\sin\alpha}$
$\cos\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\cos\alpha$	$\dfrac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\dfrac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}}$
$\tan\alpha$	$\dfrac{\tan\alpha}{\pm\sqrt{1+\tan^2\alpha}}$	$\dfrac{1}{\pm\sqrt{1+\tan^2\alpha}}$	$\tan\alpha$	$\dfrac{1}{\tan\alpha}$
$\cot\alpha$	$\dfrac{1}{\pm\sqrt{1+\cot^2\alpha}}$	$\dfrac{\cot\alpha}{\pm\sqrt{1+\cot^2\alpha}}$	$\dfrac{1}{\cot\alpha}$	$\cot\alpha$

N.B. Quando nelle formule sopra compare $\pm$ non si intende che vadano bene entrambe le opzioni, ma il segno si sceglie in base al quadrante dove cade il secondo lato dell'angolo.

α^o	α_{rad}	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
0^o	0	0	1	0
30^o	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^o	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60^o	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90^o	$\frac{\pi}{2}$	1	0	non esiste
120^o	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135^o	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
150^o	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
180^o	π	0	-1	0
210^o	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
225^o	$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
240^o	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270^o	$\frac{3\pi}{2}$	-1	0	non esiste
300^o	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315^o	$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1
330^o	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
360^o	2π	0	1	0

Angoli associati.

$$\begin{cases} \sin(-\alpha) = -\sin\alpha \\ \cos(-\alpha) = \cos\alpha \\ \tan(-\alpha) = -\tan\alpha \end{cases} \qquad \begin{cases} \sin(\pi-\alpha) = \sin\alpha \\ \cos(\pi-\alpha) = -\cos\alpha \\ \tan(\pi-\alpha) = -\tan\alpha \end{cases} \qquad \begin{cases} \sin(\alpha+\pi) = -\sin\alpha \\ \cos(\alpha+\pi) = -\cos\alpha \\ \tan(\alpha+\pi) = \tan\alpha \end{cases}$$

$$\begin{cases} \sin(\frac{\pi}{2}-\alpha) = \cos\alpha \\ \cos(\frac{\pi}{2}-\alpha) = \sin\alpha \\ \tan(\frac{\pi}{2}-\alpha) = \cot\alpha \end{cases} \qquad \begin{cases} \sin(\frac{\pi}{2}+\alpha) = \cos\alpha \\ \cos(\frac{\pi}{2}+\alpha) = -\sin\alpha \\ \tan(\frac{\pi}{2}+\alpha) = -\cot\alpha \end{cases}$$

Formule di addizione.

$$\begin{cases} \sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha \\ \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ \tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \end{cases}$$

Formule di sottrazione

$$\begin{cases} \sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha \\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ \tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \end{cases}$$

Formule di duplicazione.

$$\begin{cases} \sin 2\alpha = 2\sin\alpha\cos\alpha \\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \\ = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1 \\ \tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} \end{cases}$$

Formule di bisezione.[16]

$$\begin{cases} \sin\frac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}} \\ \cos\frac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}} \\ \tan\frac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \pm\dfrac{\sin\alpha}{1+\cos\alpha} \end{cases}$$

Formule di prostaferesi.

$$\begin{cases} \sin p \pm \sin q = 2\sin\frac{p\pm q}{2} \quad \cos\frac{p\mp q}{2} \\ \cos p + \cos q = 2\cos\frac{p+q}{2} \quad \cos\frac{p-q}{2} \\ \cos p - \cos q = -2\sin\frac{p+q}{2} \quad \sin\frac{p-q}{2} \end{cases}$$

Formule di Werner.([17])

$$\begin{cases} \sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \\ \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)] \\ \sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \end{cases}$$

Espressioni di $\sin\alpha$ e $\cos\alpha$ come funzioni razionali di $\tan\frac{\alpha}{2}$.

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}; \quad \cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$