ES 1 $\quad A = \{2D, 8R\}, \quad B = \{5D, 5R\}$

(i) X assume valori $\cancel{0}, 1, 2, 3$

$$P(X=k) = \frac{\binom{8}{k}\binom{2}{3-k}}{\binom{10}{3}}$$

$k=1 \qquad P(X=1) = \frac{8}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} \cdot 3 = \frac{1}{15}$

$k=2 \qquad P(X=2) = \frac{7}{15}$

$k=3 \qquad P(X=3) = \frac{7}{15}$

$$E[X] = \sum_{x_i} x_i \, P(X=x_i) = 1 \cdot \frac{1}{15} + 2 \cdot \frac{7}{15} + 3 \cdot \frac{7}{15} = \frac{12}{5}$$

(ii) $P(A) = P(B) = \frac{1}{2}$

X = # blocchi regolari estratti

$$P(A \mid X=3) = \frac{P(X=3 \mid A)\, P(A)}{P(X=3 \mid A)\, P(A) + P(X=3 \mid B)\, P(B)} =$$

$P(X=3 \mid A) = \frac{7}{15}$

$$P(X=3 \mid B) = \frac{\binom{5}{3}}{\binom{10}{3}} = \frac{1}{12}$$

$$= \frac{\frac{7}{15} \cdot \frac{1}{2}}{\frac{7}{15} \cdot \frac{1}{2} + \frac{1}{12} \cdot \frac{1}{2}} = \frac{28}{33}$$

ES. 2

$$f(x) = \begin{cases} 3x^c, & 0 < x < 1 \\ 0, & \text{alt.} \end{cases}, \quad c > 0$$

(i) • $f(x) \geq 0 \ \forall x$, f integ., $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\Downarrow$ Sempre verif. $\quad \Downarrow$ Sempre verif. $\quad \Downarrow$ $\int_0^1 3x^c\,dx = 1$

$$\int_0^1 3x^c\,dx = \frac{3\,x^{c+1}}{c+1}\Big|_0^1 = \frac{3}{c+1} = 1 \Rightarrow c = 2$$

$$E[X] = \int_{-\infty}^{+\infty} x\,f(x)\,dx = \int_0^1 3x^3\,dx = \frac{3}{4}x^4\Big|_0^1 = \frac{3}{4}$$

$$Var(X) = E[X^2] - E[X]^2 = \int_0^1 3x^4\,dx - \frac{9}{16} = \frac{3}{5} - \frac{9}{16} = \frac{3}{80}$$

(ii) $X_1, \dots, X_{60}$, $Z = X_1 + \dots + X_{60}$

$$P\{Z \leq 42\} = P\left\{\frac{X_1 + X_2 + \dots + X_{60} - 60 \cdot E[X_i]}{\sqrt{60 \cdot Var(X_i)}} \leq \frac{42 - 60 \cdot E[X_i]}{\sqrt{60 \cdot Var(X_i)}}\right\}$$

$$\sim P\left\{Y \leq \frac{42 - 45}{\sqrt{9/4}}\right\} = P\left\{Y \leq -3 \cdot \frac{2}{3}\right\} = P\{Y \leq -2\} =$$

$Y \sim N(0,1)$

$$= \Phi(-2) = 1 - \Phi(2) \sim 1 - 0.97725 = 0.02275$$

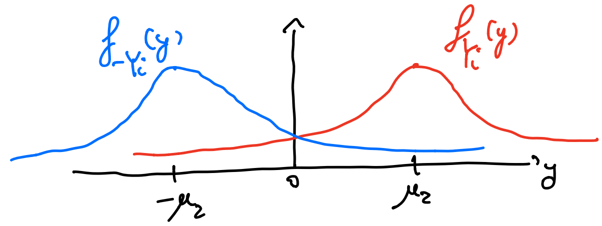
ES. 3 $\qquad X_i \sim N(\mu_1, \sigma_1^2) \qquad Y_i \sim N(\mu_2, \sigma_2^2)$, $i = 1, \dots, 6$

$Z_i = X_i - Y_i$ che assume valori nel campione statistico

$$5,\ 4,\ 3,\ 8,\ 0,\ -2$$

Z_i indipendenti

(i) $\qquad -Y_i \sim N(-\mu_2, \sigma_2^2)$



$$Z_i = X_i + (-Y_i) = N(\mu_1 + (-\mu_2),\ \sigma_1^2 + \sigma_2^2) = N(\mu_1 - \mu_2,\ \sigma_1^2 + \sigma_2^2)$$

(ii) $\qquad \mu_1, \mu_2, \sigma_1, \sigma_2$ non note

$H_0)$ $\mu_1 - \mu_2 = \mu_Z = 0\ (= \mu_0)$, $H_1)$ $\mu_1 - \mu_2 \neq 0$

$\alpha = 0.05$, $1 - \frac{\alpha}{2} = 0.975$, $n = 6$

$$C = \left\{ \frac{\sqrt{n}}{S} |\bar{Z} - \mu_0| > t_{(1-\alpha/2,\ n-1)} \right\}$$ Regione critica

$$\bar{Z} = \frac{1}{6} \sum_{i=1}^{6} z_i = 3$$

$$\bar{\sigma}^2 = \frac{1}{5}\left(\sum_{i=1}^{6} z_i^2 - 6\bar{z}^2 \right) = \frac{64}{5}$$

$$t_{(0.975,\ 5)} \sim 2.5706$$

L'ipotesi è accettabile al livello 0.05 se

$$\frac{\sqrt{6}}{\bar{\sigma}} |\bar{Z} - 0| \le t_{(0.975,5)} \iff \overset{\sim 2.05396}{\frac{\sqrt{6}}{\sqrt{64/5}}\, 3} \le 2.5706 \qquad \underline{OK}$$

(iii) $\quad H_0)\ \mu_1 - \mu_2 = \mu_0 \qquad H_1)\ \mu_1 - \mu_2 \neq 0$

Con i dati di sopra, voglio $\bar{\alpha} \geq 0.3$.

$$\bar{\alpha} = 2\left[1 - F_{T(n-1)}\left(\frac{\sqrt{n}}{\sigma}|\bar{z} - \mu_0|\right)\right] = 0.3$$

$$F_{T(5)}\left(\frac{\sqrt{6}}{\sqrt{64/5}}|3 - \mu_0|\right) = 0.85$$

$$\frac{\sqrt{6}}{\sqrt{64/5}}|3 - \mu_0| \sim 1.1558$$

$$|3 - \mu_0| \sim 1.68815$$

$$\mu_0 \in (3 - 1.68815,\ 3 + 1.68815) =$$
$$= (1.31185,\ 4.68815)$$

ES. 3 (vecchio programma) $\qquad S = \{1, 2, 3, 4\}$

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 \\ 0 & 2/3 & 1/3 & 0 \\ 0 & 0 & 1/4 & 3/4 \end{pmatrix}$$

(i)
$$\begin{cases} (a\ b\ c\ d)\ P = (a\ b\ c\ d) \\ a + b + c + d = 1 \\ a, b, c, d \geq 0 \end{cases} \iff$$

$$\iff \begin{cases} \frac{1}{2}a + \frac{2}{3}b = a & \longrightarrow \quad a = \frac{4}{3}b \\ \frac{1}{2}a + \frac{2}{3}c = b & \\ \frac{1}{3}b + \frac{1}{3}c + \frac{1}{4}d = c & \longrightarrow \quad c = \frac{1}{2}b \end{cases}$$

$$\begin{cases} \frac{3}{4} d = d \\ a+b+c+d = 1 \end{cases} \longrightarrow d = 0$$

$$\left(\tfrac{4}{3} b,\ b,\ \tfrac{1}{2} b,\ 0\right) \quad \text{con} \quad \tfrac{4}{3} b + b + \tfrac{1}{2} b = 1$$

$$\Downarrow$$

L'unica prob. inv. è $\left(\frac{8}{17}, \frac{6}{17}, \frac{3}{17}, 0\right)$

(ii) $X_0 = (0 \ \ 0 \ \ 0 \ \ 1) \qquad X_3 = X_0 P^3$

$$X_1 = X_0 P = \left(0 \quad 0 \quad \tfrac{1}{4} \quad \tfrac{3}{4}\right)$$

$$X_2 = X_1 P = \left(0 \quad \tfrac{1}{6} \quad \tfrac{13}{48} \quad \tfrac{9}{16}\right)$$

$$X_3 = \left(* \quad * \quad \underset{\displaystyle \frac{55}{192}}{\underset{\shortparallel}{P(X_3 = 3 \mid X_0 = 4)}} \quad *\right) = X_2 P =$$

$$= \left(* \quad * \quad \frac{55}{192} \quad *\right)$$

(iii) $X_0 = (1 \ \ 0 \ \ 0 \ \ 0)$

Determinare $T = \min\limits_{n \geq 1} P(X_n = 4 \mid X_0 = 1) > 0$

$$X_1 = X_0 P = \left(\tfrac{1}{2} \quad \tfrac{1}{2} \quad 0 \quad 0\right)$$

$$X_2 = X_1 P = (* \quad * \quad * \quad 0)$$

$$X_3 = X_2 P = (* \quad * \quad * \quad 0)$$

$$\nexists\, T$$