## ES. 1

Variabile aleatoria binomiale $B(1,p)$

$$P\{X=1\}=p \qquad P\{X=0\}=1-p$$

$$\{\text{Motore guasto}\} \Leftrightarrow \{X=1\}$$

a) Aereo bimotore non vola se e solo se

$$\{X_1=1,\ X_2=1\}$$

$$P\{X_1=1, X_2=1\} = P\{X_1=1\}\,P\{X_2=1\} = p^2$$

Aereo quadrimotore non vola se e solo se almeno tre motori sono guasti.

$$X_1X_2X_3X_4 = \{0111,\ 1011,\ 1101,\ 1110,\ 1111\}$$

$$P\{\text{Aereo non voli}\} = 4p^3(1-p) + p^4 = $$
$$= 4p^3 - 3p^4$$

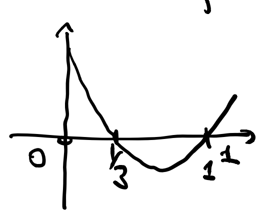
b) $P\{\text{Bimotore non voli}\} < P\{\text{Quadrim. non voli}\}$

$$\Updownarrow$$

$$p^2 < 4p^3 - 3p^4$$

$$\Leftrightarrow \quad 3p^4 - 4p^3 + p^2 < 0 \quad \Leftrightarrow \quad p^2(3p^2 - 4p + 1) < 0$$

$$\Leftrightarrow \quad 3p^2 - 4p + 1 < 0 \quad \Leftrightarrow \quad p \in \underbrace{(1/3,\, 1)}_{\text{Bim. più sicuri}}$$



ES. 2

$$f(x) = \begin{cases} c x^2, & -1 < x < 1 \\ 0, & \text{alt.} \end{cases}$$

a) $f(x) \geq 0$ , $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\Updownarrow$ $\qquad\qquad \Updownarrow$

$c \geq 0$ , $\int_{-1}^{1} c x^2 \,dx = 1 \iff c \frac{x^3}{3}\Big|_{-1}^{1} = 1$

$\iff \frac{2}{3} c = 1 \iff c = \frac{3}{2}$

$$F(x) = P\{X \leq x\} = \int_{-\infty}^{x} f(t)\,dt$$

$$F(x) = \begin{cases} 0, & x \leq -1 \\ \frac{x^3}{2} + \frac{1}{2}, & -1 \leq x \leq 1 \\ 1, & x \geq 1 \end{cases}$$

$$\int_{-1}^{x} f(t)\,dt = \int_{-1}^{x} \frac{3}{2} t^2 \,dt = \frac{x^3}{2} + \frac{1}{2}$$

b) $r_\beta$ è il $\beta$-quantile $\iff F(r_\beta) = \beta$

$\iff \frac{r_\beta^3}{2} + \frac{1}{2} = \beta \iff r_\beta = (2\beta - 1)^{1/3}$

$n \in \mathbb{N}$, $E[X^n] = \int_{-\infty}^{+\infty} x^n f(x)\,dx = \int_{-1}^{1} \frac{3}{2} x^{n+2}\,dx =$

$$= \frac{3}{2(n+3)} x^{n+3}\Big|_{-1}^{1} = \begin{cases} 0, & n \text{ dispari} \\ \frac{3}{n+3}, & n \text{ pari} \end{cases}$$

c) $Y_n = 2 + X^n$

$$E[Y_n] = 2 + E[X^n] = \begin{cases} 2, & n \text{ dispari} \\ 2 + \frac{3}{n+3}, & n \text{ pari} \end{cases}$$

$$\text{Var}(Y_n) = \text{Var}(2 + X^n) = \text{Var}(X^n) =$$

$$\text{Var}(X+a) = E[(X+a - E[X+a])^2]$$
$$= E[(X - E[X])^2] = \text{Var}(X)$$

$$= E[(X^n)^2] - E[X^n]^2 = E[X^{2n}] - E[X^n]^2$$

$$= \begin{cases} \frac{3}{2n+3} - 0 = \frac{3}{2n+3}, & n \text{ dispari} \\ \frac{3}{2n+3} - \left(\frac{3}{n+3}\right)^2, & n \text{ pari} \end{cases}$$

$$E[Y_n] \xrightarrow[n \to +\infty]{} 2, \qquad \text{Var}(Y_n) \xrightarrow[n \to +\infty]{} 0$$

$\Rightarrow$ $Y_n$ converge a 2 in probabilità.

$\longleftrightarrow$

<u>ES. 3</u> $X_1, \dots, X_{64}$ v.a. $N(m, \sigma^2)$ $\sigma$ non nota

$$\bar{x} = 47.1, \quad \bar{\sigma} = 2.4$$

a) $H_0)$ $m \geq 48$, $H_1)$ $m < 48$

$C = \left\{ \frac{\sqrt{n}}{S}(\bar{X} - 48) < t_{(\alpha, 63)} \right\}$ regione critica al livello $\alpha$

$$\bar{\alpha} = F_{T(63)}\left(\frac{\sqrt{n}}{\bar{\sigma}}(\bar{x}-48)\right) = F_{T(63)}\left(\frac{\sqrt{64}}{2.4}(47.1-48)\right)$$

$$\overset{R}{\sim} 0.00193$$

$$\sim \Phi\left(\frac{\sqrt{64}}{2.4}(47.1-48)\right) \sim \Phi(-3) = 1-\Phi(3)$$

$$\sim 1 - 0.99865 = 0.00135$$

b) Fissando $n=64$, $\bar{\sigma}=2.4$, $\mu_0=48$
per quale $\bar{x}$ abbiamo $\bar{\alpha}=0.3$ ?

$$\bar{\alpha} = F_{T(63)}\left(\frac{\sqrt{64}}{2.4}(\bar{x}-48)\right) = 0.3$$

$$\Leftrightarrow \quad \frac{\sqrt{64}}{2.4}(\bar{x}-48) = \tau_{(0.3,63)}$$

$$\tau_{(0.3,63)} \overset{R}{\sim} -0.527$$

$$\tau_{(0.3,63)} \sim q_{0.3} = -q_{0.7} \sim -0.53$$

$$\Rightarrow \quad \tilde{x} \sim 47.84$$

## ES. 3 vecchio programma

$S = \{1, 2, 3\}$

$$P = \begin{pmatrix} 1/2 & ? & 0 \\ 0 & 0 & 1 \\ ? & 2/3 & 0 \end{pmatrix}$$

a) $p_{ij} \geq 0, \quad \sum_{j=1}^{3} p_{ij} = 1 \quad \forall i$

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/3 & 2/3 & 0 \end{pmatrix}$$

b) $P\{X_{n+1} = i\} = \sum_{j=1}^{3} P\{X_{n+1} = i \mid X_n = j\} \, P\{X_n = j\}$

$= \sum_{j=1}^{3} p_{ji} \, P\{X_n = j\}$

$X_{n+1} = X_n \, P$

$X_0 = (1 \quad 0 \quad 0)$

$X_1 = (1 \quad 0 \quad 0) \, P = (\frac{1}{2} \quad \frac{1}{2} \quad 0)$

$X_2 = (\frac{1}{2} \quad \frac{1}{2} \quad 0) \, P = (\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{2})$

$X_3 = (\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{2}) \, P = (\frac{7}{24} \quad \frac{11}{24} \quad \frac{6}{24})$

c) $\pi P = \pi \qquad \pi = (\pi_1 \quad \pi_2 \quad \pi_3)$

$\sum_{i=1}^{3} \pi_i = 1$

$$\Longleftrightarrow \begin{cases} \frac{1}{2}\pi_1 + \frac{1}{3}\pi_3 = \pi_1 \\ \frac{1}{2}\pi_1 + \frac{2}{3}\pi_3 = \pi_2 \\ \pi_2 = \pi_3 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases} \Longleftrightarrow \pi = (\tfrac{1}{4} \quad \tfrac{3}{8} \quad \tfrac{3}{8})$$