$\boxed{\text{ES. 1}}$ $X_i = B(1,p) \quad p = \frac{7}{10} \quad p = P(X_i = 1), \quad 1-p = P(X_i = 0)$

$$Y = X_1 + \dots + X_5 = B(5,p)$$

(a) App. funz. $\Leftrightarrow Y \geq 2$

$$P(Y=0) = \binom{5}{0} p^0 (1-p)^5 = \left(\frac{3}{10}\right)^5$$

$$P(Y=1) = \binom{5}{1} p^1 (1-p)^4 = 5 \cdot \frac{7}{10} \cdot \left(\frac{3}{10}\right)^4$$

$$P(Y \geq 2) = 1 - P(Y=0) - P(Y=1) = 1 - \frac{3^5}{10^5} - \frac{5 \cdot 7 \cdot 3^4}{10^5} =$$

$$= 10^{-5}(100000 - 243 - 2835) = \frac{96922}{100000} = \frac{48461}{50000} = 0.96922$$

(b) $$P(X_1 = 0 \mid Y \geq 2) = \frac{P(X_1 = 0 \cap Y \geq 2)}{P(Y \geq 2)} = \frac{P(Y \geq 2 \mid X_1 = 0) \cdot P(X_1 = 0)}{P(Y \geq 2)} =$$

$$= \frac{P(X_2 + X_3 + X_4 + X_5 \in \{2,3,4\}) \cdot P(X_1 = 0)}{P(Y \geq 2)}$$

- $Z = X_2 + X_3 + X_4 + X_5 = B(4,p)$

$$P(Z \geq 2) = 1 - P(Z=0) - P(Z=1) =$$

$$= 1 - \binom{4}{0} p^0 (1-p)^4 - \binom{4}{1} p^1 (1-p)^3 =$$

$$= 1 - \frac{3^4}{10^4} - 4 \cdot \frac{7 \cdot 3^3}{10^4} = 10^{-4}(10000 - 81 - 756) = \frac{9163}{10000}$$

$$\boxed{P(X_1 = 0 \mid Y \geq 2) = \frac{9163}{10000} \cdot \frac{3}{10} \cdot \frac{100000}{96922} = \frac{27489}{96922} \sim 0.283}$$

- $P(X_1 = 0 \cap Y \geq 2) = (1-p) \cdot \binom{4}{2} p^2 (1-p)^2 + (1-p)\binom{4}{3} p^3 (1-p) + (1-p)\binom{4}{4} p^4$

$$= \frac{3}{10} \cdot 6 \cdot \frac{7^2 \cdot 3^2}{10^4} + \frac{3}{10} \cdot 4 \cdot \frac{7^3 \cdot 3}{10^4} + \frac{3}{10} \cdot \frac{7^4}{10^4} =$$

$$= 10^{-5}(7938 + 12348 + 7203) = 10^{-5} \cdot 27489$$

$$\boxed{P(X_1 = 0 \mid Y \geq 2) = \frac{27489}{100000} \cdot \frac{100000}{96922} \sim 0.283}$$

$\boxed{\text{ES. 2}}$

$$F(x) = \begin{cases} 0, & x < 0 \\ ax + bx^2, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases} \quad , \quad a, b \in \mathbb{R}$$

(a) • $\lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to +\infty} F(x) = 1$ ✓

• F è deb. monotona crescente

$\forall x \in (0,1), 1 \ge F(x) \ge 0 \ , \quad F'(x) \ge 0$

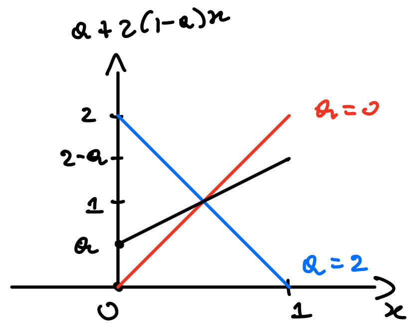
$(1 \ge x(a+bx) \ge 0), \quad a + 2bx \ge 0$

• F è continua

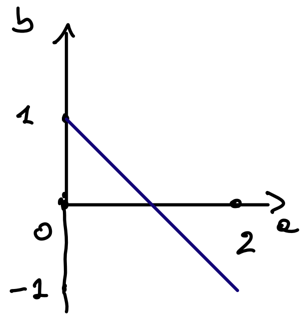
$F(0) = 0, \quad F(1) = 1 \iff a + b = 1$

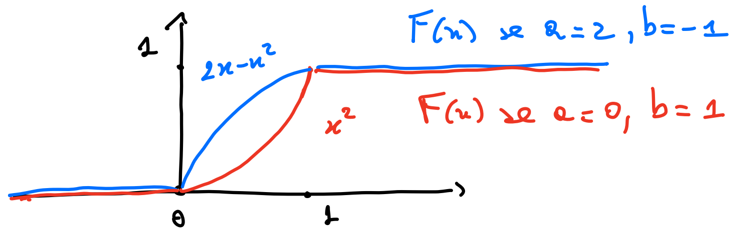
$\Rightarrow \quad a + 2bx = a + 2(1-a)x \ge 0 \quad \forall x \in (0,1)$

$\Rightarrow \quad a \ge 0 \ , \ 2 - a \ge 0$




$\Rightarrow \quad a \in [0,2] \ , \ b = 1 - a$




(b) $f(x) = \begin{cases} a + 2bx, & 0 < x < 1 \\ 0, & \text{alt.} \end{cases} \quad , \quad a \in [0,2], \ b = 1 - a$

$$E[X] = \int_0^1 x f(x)\,dx = \int_0^1 (ax + 2bx^2)\,dx = \left(\tfrac{1}{2}ax^2 + \tfrac{2}{3}bx^3\right)\Big|_0^1 =$$

$$= \tfrac{1}{2}a + \tfrac{2}{3}b = \tfrac{1}{2}a + \tfrac{2}{3}(1-a) = \tfrac{2}{3} + \left(\tfrac{1}{2} - \tfrac{2}{3}\right)a = \tfrac{2}{3} - \tfrac{1}{6}a$$

• $\frac{2}{3} - \frac{1}{6}a = \frac{1}{2} \iff \frac{1}{6}a = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \iff a = 1$

• $\frac{2}{3} - \frac{1}{6}a = -1 \iff \frac{1}{6}a = \frac{2}{3} + 1 = \frac{5}{3} \iff a = 10$ NO

(c) $f_Y(y) = \begin{cases} f_X(h^{-1}(y)) \left| \frac{d}{dy} h^{-1}(y) \right|, & y \in h((0,1)) \\ 0, & \text{alt.} \end{cases}$

$h(t) = \log t$ , $h(0,1) = (-\infty, 0)$ , $h^{-1}(y) = e^y$ $\forall y \in (-\infty, 0)$

$f_Y(y) = \begin{cases} 0, & y > 0 \\ (a + 2b e^y) e^y, & y \le 0 \end{cases}$ , $b = 1-a$, $a \in [0,2]$

**ES. 3**

$X_1, \dots, X_n$ $n = 81$ , $\bar{t} = 235.44$ , $\bar{\sigma} = 3.6$

$t = 231.06$

(a) Precisione della stima = 0.82

$\tau_{(1-\alpha/2,\, n-1)} \frac{\bar{\sigma}}{\sqrt{n}} = 0.82 \iff \tau_{(1-\alpha/2,\, 80)} = \frac{9}{3.6} \cdot 0.82 = 2.05$

$\Rightarrow 1 - \alpha/2 \sim 0.98 \iff \frac{\alpha}{2} \sim 0.02 \iff \alpha \sim 0.04$

Livello di fiducia $\sim 0.96$

(b) $H_0)$ $\sigma^2 \le 10 = \sigma_0^2$ test unilatero sulla varianza

Regione critica $C = \left\{ \frac{(n-1)\,\bar{\sigma}^2}{\sigma_0^2} > \chi^2_{(1-\alpha,\, n-1)} \right\}$

p-value $\bar{\alpha} = 1 - F_{\chi^2(n-1)}\left( \frac{(n-1)}{\sigma_0^2}\, \bar{\sigma}^2 \right) =$

$= 1 - F_{\chi^2(80)}\left( \frac{80}{10}\, 3.6^2 \right) =$

$= 1 - F_{\chi^2(80)}(103.68) \sim 1 - 0.96 = 0.04$

**ES. 3 BIS**

$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ \lambda & 0 & 1-\lambda \\ 0 & 2/3 & 1/3 \end{pmatrix}$

(a) $X_0 = (1 \quad 0 \quad 0)$ $X_1 = (1/2 \quad 1/2 \quad 0)$

$$X_2 = \begin{pmatrix} \frac{1}{4}+\frac{1}{2}\lambda & \frac{1}{4} & \frac{1}{2}-\frac{1}{2}\lambda \end{pmatrix}$$

$$\frac{1}{4} \geq \frac{1}{2} - \frac{1}{2}\lambda \iff \frac{1}{2}\lambda \geq \frac{1}{4} \iff \lambda \geq \frac{1}{2}$$

(b) $$X_3 = \begin{pmatrix} * & \underbrace{\frac{1}{8}+\frac{1}{4}\lambda + \frac{1}{3} - \frac{1}{3}\lambda}_{\frac{11}{24} - \frac{1}{12}\lambda} & * \end{pmatrix}$$

$a+b+c=1$

(c) $$\begin{cases} \frac{1}{2}a + \lambda b = a \\ \frac{1}{2}a + \frac{2}{3}c = b \\ (1-\lambda)b + \frac{1}{3}c = c \end{cases} \qquad P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ \lambda & 0 & 1-\lambda \\ 0 & 2/3 & 1/3 \end{pmatrix}$$

$$\begin{cases} a = 2\lambda b \\ c = \frac{3}{2}(1-\lambda)b \\ a+b+c=1 \end{cases}$$

$$b\left(2\lambda + \frac{3}{2} - \frac{3}{2}\lambda + 1\right) = 1$$

$$b\left(\frac{1}{2}\lambda + \frac{5}{2}\right) = 1$$

$$b = \frac{2}{\lambda+5}$$

$$(a,b,c) = \left(\frac{4\lambda}{\lambda+5}, \frac{2}{\lambda+5}, \frac{3(1-\lambda)}{\lambda+5}\right)$$

$$(a,b,c) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \iff \begin{cases} \frac{4\lambda}{\lambda+5} = \frac{1}{3} \\ \frac{2}{\lambda+5} = \frac{1}{3} \\ \frac{3(1-\lambda)}{\lambda+5} = \frac{1}{3} \end{cases} \qquad \nexists \lambda$$