Ist. Mat. I - C/A

14/12/22

(14) $\displaystyle\sum \frac{n\,2^n}{e^{n/2}}$

$$\sqrt[n]{a_n} = \frac{\sqrt[n]{n}\cdot 2}{\sqrt{e}} \to \frac{2}{\sqrt{e}} > 1 \qquad \sqrt[n]{n} = \exp\left(\log \sqrt[n]{n}\right) = \exp\left(\frac{1}{n}\cdot\log(n)\right) \to \exp(0) = 1$$

diverge a $+\infty$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{(n+1)2^{n+1}}{e^{(n+1)/2}}}{\dfrac{n\cdot 2^n}{e^{n/2}}} = \frac{n+1}{n}\cdot\frac{2}{e^{1/2}} \longrightarrow \frac{2}{\sqrt{e}}$$

(15) $\displaystyle\sum \frac{1}{\sqrt{n^3-n}}$

$$\frac{1}{\sqrt{n^3-n}} \simeq \frac{1}{n^{3/2}} \qquad 3/2 > 1 \Rightarrow \text{converge per confronto con } \sum \frac{1}{n^\alpha},\ \alpha = 3/2 > 1$$

(16) $\displaystyle\sum \frac{n+\log(n)}{(n+\cos(n))^3}$

$$\simeq \frac{1}{n^2} \Rightarrow \text{converge}$$

(17) $\displaystyle\sum \frac{\log(n)}{n^2}$

$$\frac{\log(n)}{n^2}\ \cancel{\not\simeq}\ \frac{1}{n^2} \qquad \frac{\log(n)}{n^2} < \frac{1}{n^{2-\varepsilon}} \quad \forall \varepsilon > 0 \ (\text{per } n >> 0)$$

(ad es $\varepsilon = 1/2$: confronto con $\sum \frac{1}{n^\alpha}$ $\alpha = 3/2 > 1$)
converge.

(18) $\sum \frac{2^n+1}{3^n+1}$ $\qquad \frac{2^n+1}{3^n+1} \simeq \left(\frac{2}{3}\right)^n$ converge

Pearson p. 185

$\boxed{27}$ ① $\sum \frac{2+\log^3(n)}{n^2-2}$

$$a_n \cong \frac{\log^3(n)}{n^2} < \frac{1}{n^\alpha} \quad \forall \alpha < 2$$

② $\sum \frac{3}{n+\log(n)}$ $\qquad a_n \cong \frac{1}{n}$ diverge

③ $\sum \frac{\sqrt{n}+\sin(n)}{n+2}$ $\qquad a_n \cong \frac{1}{\sqrt{n}}$

$\sum \frac{1}{n^\alpha}$ $\qquad \alpha = \frac{1}{2} < 1 \Rightarrow$ diverge

④ $\sum \frac{\sqrt{n}+\cos(n)}{n^2-n+1}$ $\qquad a_n \cong \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$

$\sum \frac{1}{n^\alpha}$ $\qquad \alpha = \frac{3}{2} > 1 \Rightarrow$ converge

⑤ $\sum \frac{n+\log^3(n)}{\sqrt[3]{n^5}}$ $\qquad a_n \cong \frac{n}{n^{5/3}} = \frac{1}{n^{5/3-1}} = \frac{1}{n^{2/3}}$

$$\sum \frac{1}{n^\alpha} \qquad \alpha = \frac{2}{3} < 1 \implies \text{diverge}$$

(6) $$\sum \frac{n + \log^3(n)}{\sqrt[3]{n^7}} \qquad a_n \cong \frac{n}{n^{7/3}} = \frac{1}{n^{7/3-1}} = \frac{1}{n^{4/3}}$$

$$\sum \frac{1}{n^\alpha} \qquad \alpha = \frac{4}{3} > 1 \implies \text{converge}$$

(28) (1) $$\sum \frac{3 + \sin(n)}{\sqrt{n} + 2^n}$$

$$a_n \cong \frac{1}{2^n} \implies \text{converge}$$

il rapporto rimane limitato lontano da 0

(2) $$\sum \frac{e^n}{2^n + 3^n} \qquad a_n \cong \frac{e^n}{3^n} = \left(\frac{e}{3}\right)^n$$

$$\frac{e}{3} < 1 \implies \text{converge}$$

(3) $$\sum \frac{n + e^{-n}}{n^2} \qquad a_n \cong \frac{1}{n} \qquad \text{diverge}$$

(4) $$\sum \frac{n \cdot e^{-n}}{n^2 + 1} \qquad a_n \cong \frac{1}{n} e^{-n} \leq e^{-n} \qquad \text{converge}$$

⑤ $\sum \frac{e^n}{\sinh(n)}$

$a_n = \frac{e^n}{\frac{e^n - e^{-n}}{2}} = \frac{2e^n}{e^n - e^{-n}} \to 2$

$\Rightarrow$ diverge

⑥ $\sum \frac{(n!)^2}{(2n)!}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{((n+1)!)^2}{(2n+2)!}}{\frac{(n!)^2}{(2n)!}}$$

$$= \frac{\frac{(n+1)^2 \cdot \cancel{(n!)^2}}{(2n+2)(2n+1)\cdot \cancel{(2n)!}}}{\frac{\cancel{(n!)^2}}{\cancel{(2n)!}}} = \frac{(n+1)^2}{(2n+2)(2n+1)} \to \frac{1}{4}$$

$\Rightarrow$ converge $\left(\frac{1}{4} < 1\right)$

Zanichelli p. 193

(49) Qual'è la forma ottimale di una lattina da 33 cl?

Uso unità di misura cm.

$33\,cl = 33 \cdot \frac{1}{100}\,l = 33 \cdot \frac{1}{100}\,(dm)^3 = 33 \cdot \frac{1}{100} \cdot (10cm)^3$

$= 330\,cm^3$

Suppongo lattina = cilindro di raggio di base r cm e altezza h cm.

Volume $= 330 \text{ cm}^3$

$$\pi r^2 \cdot h \cdot \text{cm}^3 = 330 \text{ cm}^3$$

$$\Longrightarrow h = \frac{330}{\pi} \cdot \frac{1}{r^2}$$

superficie in cm^2 :

$$2 \cdot \pi r^2 + 2\pi r \cdot h = 2\pi r^2 + 2\pi r \cdot \frac{330}{\pi} \cdot \frac{1}{r^2}$$

$$= 2\pi r^2 + 660 \cdot \frac{1}{r}$$

$$S(r) = 2\pi r^2 + 660 \cdot \frac{1}{r}$$

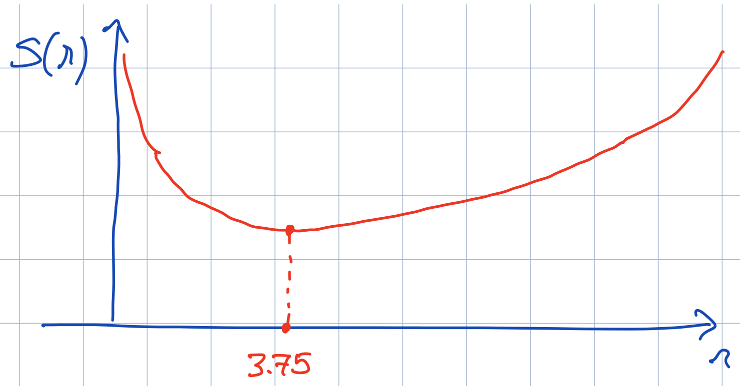
$$\lim_{r \to +\infty} S(r) = \lim_{r \to 0^+} S(r) = +\infty$$

Cerchiamo $\min\{S(r) : r > 0\}$. Calcolo

$$S'(r) = 4\pi r - \frac{660}{r^2}$$

$$S'(r) = 0 \iff 4\pi r = \frac{660}{r^2}$$

$$\iff r = \sqrt[3]{\frac{660}{4\pi}} \simeq 3.75$$



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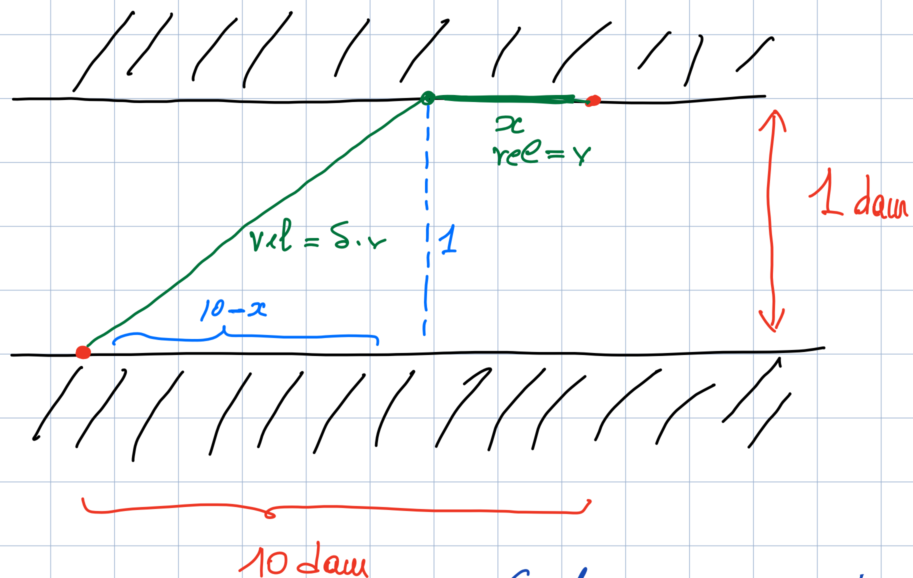


Cerchiamo x che rende minimo il tempo di percorrenza.

Dovremo avere: $\lim_{\delta \to 1} x = 0$; $\lim_{\delta \to 0} x = 10$.

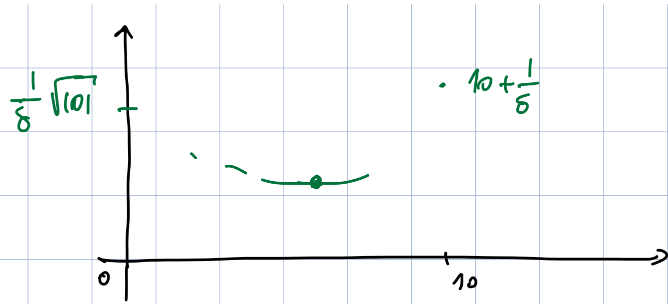
tempo impiegato:

(vel = $\frac{\text{spazio}}{\text{tempo}}$

$\Rightarrow$ tempo = $\frac{\text{spazio}}{\text{vel}}$)

$$\frac{x}{v} + \frac{\sqrt{1+(10-x)^2}}{\delta \cdot v}$$

$$T(x) = x + \frac{1}{\delta}\sqrt{1+(10-x)^2} \qquad x \in [0,10]$$



$$T'(x) = 1 + \frac{1}{\delta} \cdot \frac{\cancel{2} \cdot (10-x) \cdot (-1)}{\cancel{2} \cdot \sqrt{101 - 20x + x^2}}$$

$$T'(x) = 0 \iff$$

$$\frac{1}{\delta} \cdot \frac{10-x}{\sqrt{101-20x+x^2}} = 1$$

$$10 - x = \delta \cdot \sqrt{101 - 20x + x^2}$$

$$x^2 - 20x + 100 = \delta^2 (x^2 - 20x + 101)$$

$$(1-\delta^2)x^2 - 20(1-\delta^2)x + 100 - 101\delta^2 = 0$$

$$x^2 - 20x + \frac{100 - 101\delta^2}{1-\delta^2} = 0$$

$$x = 10 - \sqrt{100 - \frac{100 - 101\delta^2}{1-\delta^2}}$$

$$= 10 - \sqrt{\frac{100 - 100\delta^2 - 100 + 101\delta^2}{1-\delta^2}}$$

$$= 10 - \frac{\delta}{\sqrt{1-\delta^2}}$$

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Esercizio (compito prova 6/12/22):

$$f(x) = x \cdot e^{\frac{1}{x^2}}$$

(A) Trovare più grande $D \subset \mathbb{R}$ t.c. $f : D \to \mathbb{R}$

$D = \mathbb{R} \setminus \{0\}$.

$$\lim_{x \to 0^+} x \cdot e^{1/x^2} = +\infty \qquad \lim_{x \to 0^-} x \cdot e^{\frac{1}{x^2}} = -\infty$$

(B) Trovare tutti gli asintoti del grafico di f.

Asintoto verticale $x = 0$

$$\lim_{x \to \pm\infty} x \cdot e^{1/x^2} = \pm\infty$$

$$\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} e^{\frac{1}{x^2}} = 1$$

Potenziali asintoti obliqui $y = x + c$

$$\lim_{x \to \pm\infty} \left(f(x) - 1 \cdot x \right) = \lim_{x \to \pm\infty} x \cdot \left(e^{\frac{1}{x^2}} - 1 \right) \qquad \infty \cdot 0$$

$$= \lim_{x \to \pm\infty} x \cdot \left(1 + \frac{1}{x^2} + o\left(\frac{1}{x^2}\right) - 1 \right)$$

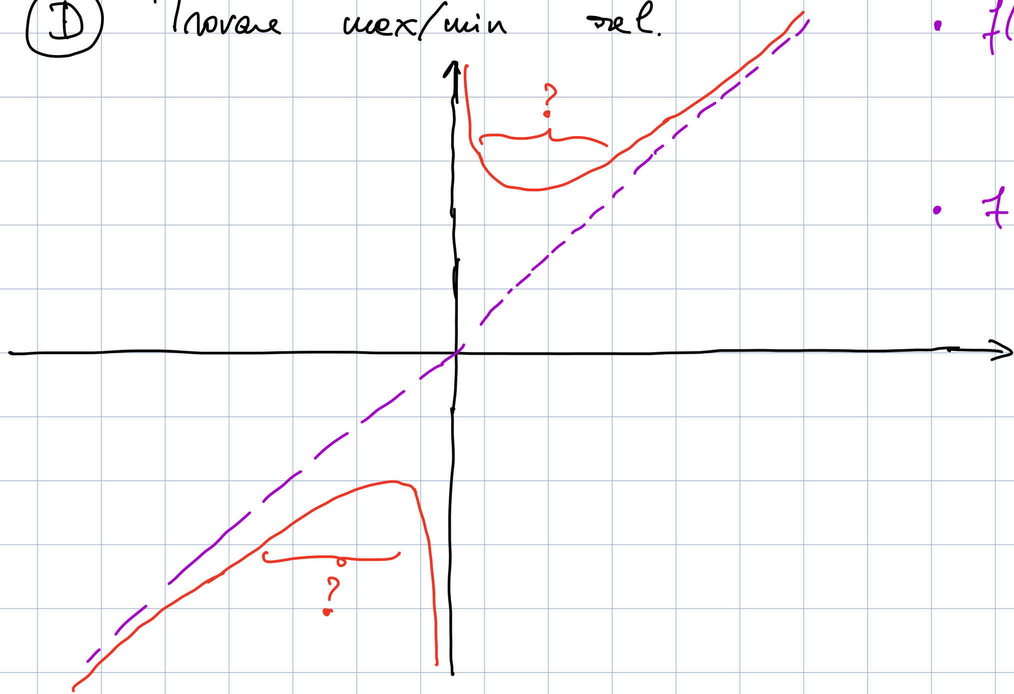
$$= \lim_{x \to \pm\infty} \left(\frac{1}{x} + o\left(\frac{1}{x}\right) \right) = 0$$

$\Longrightarrow$ asinto obliquo $y = x$ dx/sin.

(C) Trovare gli zeri di f.

$x \cdot e^{1/x^2} = 0$ nessuno

Ⓓ Trovare max/min rel.



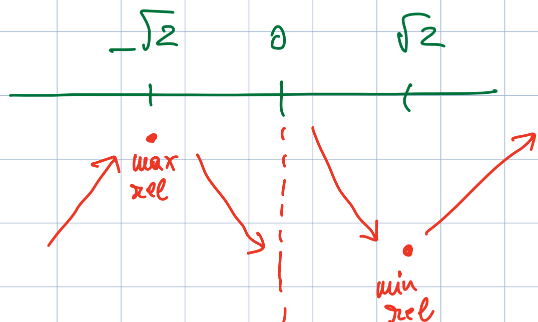
- $f(-x) = -x \cdot e^{\frac{1}{(-x)^2}} = -x \cdot e^{\frac{1}{x^2}} = -f(x)$ dispari
- $f(x)$ concorde con x

$$f(x) = x \cdot e^{\frac{1}{x^2}}$$

$$f'(x) = e^{\frac{1}{x^2}} + x \cdot e^{\frac{1}{x^2}} \cdot \left(-\frac{2}{x^3}\right)$$

$$= \boxed{e^{\frac{1}{x^2}} \left(1 - \frac{2}{x^2}\right)}$$

stesso segno di $1 - \frac{2}{x^2}$ cioè di $x^2 - 2$



Non richiesto: concavità/convessità:

$$f''(x) = e^{\frac{1}{x^2}} \cdot \left(-\frac{2}{x^3}\left(1 - \frac{2}{x^2}\right) + \frac{4}{x^3}\right)$$

$$= e^{\frac{1}{x^2}}\left(-\frac{2}{x^3}+\frac{4}{x^5}+\frac{4}{x^3}\right)$$

$$= e^{\frac{1}{x^2}}\left(\frac{2}{x^3}+\frac{4}{x^5}\right) = \underbrace{\frac{2}{x^5}\cdot \underbrace{e^{\frac{1}{x^2}}\cdot(x^2+2)}_{>0}}_{\text{concorde con } x}$$

$\Longrightarrow$ f concava su $(-\infty,0)$

f convessa su $(0,+\infty)$

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Compito prova 25/11/22 — quesiti:

① $A=\{0,\dots,6\}$

$B=\{0,\dots,4\}$

$f: A\to B$ $\quad f(x) =$ resto di $(2x^3+3):5$.

Iniettiva? Suriettiva?

x	$2x^3+3$	resto di $(2x^3+3):5$	
0	3	3	
1	5	0	no iniettiva
2	19	4 ✓	
3	57	2 ✓	
4	$2\cdot 64+3=131$	1 ✓	suriettiva
5	$2\cdot125+3$	3 ✓	
6	$2\cdot(\dots 6)+3=\dots 5$	0 ✓	

② risolvere $\log_2(x-3)+\log_2(x-5)>0$

$$\begin{cases} x-3>0 \\ x-5>0 \\ \log_2\left((x-3)(x-5)\right)>0 \end{cases}$$

$$\begin{cases} x>5 \\ x^2-8x+15>1 \end{cases}$$

$$\begin{cases} x>5 \\ x^2-8x+14>0 \end{cases}$$

$4\pm\sqrt{16-14}=4\pm\sqrt{2}$

$x>4+\sqrt{2}$