A New Semiconcavity Result for the Minimum Time Function

Piermarco CANNARSA
Dipartimento di Matematica
Università di Roma Tor Vergata
Via della Ricerca Scientifica 1
00133 Roma , Italy

Abstract

The Minimum Time function $T(x)$ of a nonlinear control system $\dot{x} = f(x, u)$ is the viscosity solution of a suitable boundary value problem on the so-called controllable set $\mathcal{R}$. Under a suitable controllability assumption at the boundary of the target K, $T(x)$ is known to be semiconcave in $\mathcal{R} \setminus K$ provided that the set K satisfies a suitable interior sphere condition, see [1].

Here, we present a new theorem that integrates the aforementioned results to extend the semiconcavity property to problems with a general target, such as, for example, a single point. Our method consists in showing that the controllable set in time τ—that is the set of all initial states that can be steered to K in a prescribed time $\tau > 0$—satisfies an interior sphere condition if τ is sufficiently small and the velocity set $f(x, U)$ is sufficiently smooth for $x \in \partial K$. Then, the semiconcavity of $T(x)$ for a general target follows from the optimality principle and the results of [1].

References

[1] Cannarsa P., Sinestrari C., Convexity properties of the minimum time function, *Calc. Var.* **3** (1995), 273–298.