Existence and Nonexistence of Solutions for Nonlinear Elliptic Equations With Measure Data

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Abstract

Let us consider the following quasilinear (model) problem

$$\begin{cases} -\Delta u + u\,|\nabla u|^2 = \delta_{x_0} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

with Ω a bounded open subset of $\mathbf{R}^N$, $N \geq 2$. It has been recently proved by Brezis and Nirenberg that this problem has no solution, since sets of zero capacity are removable singularities for the equation $-\Delta u + u\,|\nabla u|^2 = 0$.

We will show further evidence of this fact, by proving that (1) (and more general equations with the same structure) has no solutions obtained by approximation; i.e., approximating δ_{x_0} with a sequence of regular functions yields a sequence u_n of solutions of (1) which converges to zero. Furthermore, we will show that the zero solution is "stable" also under very singular perturbation of the data, in the sense that it is obtained as limit of a sequence of solutions of (1) with data which converge to zero in $L^1_{\text{loc}}(\Omega \setminus \{x_0\})$, without assumption on the behaviour near x_0.

The same phenomenon will be shown for solutions of problems such as

$$\begin{cases} -\Delta u + |u|^{q-1}\,u = \delta_{x_0} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with q large enough.