Starshaped Compact Hypersurfaces With a Prescribed Function of Principal Curvatures

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Let $\mathcal{R}^{n+1}(K)$, $n \geq 2$, $K = -1, 0, 1$, be a Riemannian manifold of sectional curvature K and m an integer, $1 \leq m \leq n$. We are interested in establishing conditions for existence of a smooth hypersurface M in $\mathcal{R}^{n+1}(K)$ which is starshaped relative to some point $O \in \mathcal{R}^{n+1}(K)$ and whose (normalized) elementary symmetric function H_m of principal curvatures of M satisfies the equation

$$H_m = \psi_{|M}, \tag{1}$$

where ψ is a given function in $\mathcal{R}^{n+1}(K)$.

In analytic formulation this problem reduces to a nonlinear, second order elliptic equation of Monge-Ampère type on a unit sphere S^n. In Euclidean space $R^{n+1}(= \mathcal{R}^{n+1}(0))$, earlier results by I. Bakelman and B. Kantor and by A. Treibergs and W. Wei give such conditions when $m = 1$ (the mean curvature case), by V. Oliker when $m = n$ (the Gauss curvature case), and by L. Caffarelli, L. Nirenberg and J. Spruck when $1 < m < n$. For $K \neq 0$ and $m = n$ the problem was investigated by V. Oliker.

In this talk we will discuss the same problem in spaces $\mathcal{R}^{n+1}(K)$ with $K \neq 0$. We will present results showing that if conditions on the behavior of the function ψ are tied together with the behavior of the metric of $\mathcal{R}^{n+1}(K)$ then it is possible to obtain some a priori estimates for solutions of equation (1). As a consequence, we establish conditions for existence of solutions to (1) for any m, $1 \leq m \leq n$, and $K = 1$.

The presented results are obtained jointly with João Lucas M. Barbosa and Jorge Lira from University of Cearà, Brazil and YanYan Li from Rutgers University, USA.