A Modification of Frankl's Problem Associated with the Airfoil Design

A.G.Kuz'min
St. Petersburg State University, Russia
e-mail alexander.kuzmin@pobox.spbu.ru

Frankl (1947) considered transonic flow with local supersonic regions over a symmetric airfoil and suggested physical arguments in favour of a problem in which the Neumann condition was prescribed on the axis of symmetry and on the airfoil except for an arc Γ_a. The uniqueness of the smooth solution to Frankl's problem was established by Morawetz (1956), who employed the hodograph plane and used an auxiliary function that was monotonous along the characteristics in the supersonic region. Cook (1978) proved a similar theorem directly in the physical plane. The important consequence of the uniqueness is that if a smooth solution to the Frankl problem exists, then it determines uniquely the shockless shape of the arc Γ_a. Thus, the Frankl problem is actually associated with airfoil adaptation to changing flow conditions.

In this work, we study a modification of the Frankl problem in which the arc Γ_a is free of boundary conditions, and the Neumann condition is replaced by the Dirichlet one along a portion of the airfoil. We demonstrate that the above mentioned result on the uniqueness remains valid in this case. From the practical point of view, the problem at hand is concerned with the shockless airfoil design under a given target velocity on a portion of the airfoil. The obtained result shows that the numerical methods, in which the slip condition / flow velocity distribution are prescribed over the full airfoil, cannot guarantee the absence of shock waves from the flow.

References

[1] Frankl F I (1947) Prikladnaya Mat i Mekh. **11**, no.1: 199–202.

[2] Morawetz C S (1956) Comm. Pure Appl. Math. **9**: 45–68.

[3] Cook L P (1978) Indiana University Math J. **27**, no. 1: 51–71.