Steady States for Quantum Hydrodynamic models: Existence of positive solutions and limiting behavior of dispersion/diffusion singular perturbations

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We analyze a quantum trajectory model given by a steady-state hydrodynamic system for quantum fluids with positive constant temperature in bounded domains for arbitrary large data. This system is a dispersive perturbation of transonic flow models. The momentum equation can be written as a dispersive third-order equation for the particle density where viscous effects may be incorporated.

The phenomena that admit positivity of the solutions are studied. The cases, one space dimensional dispersive or non-dispersive, viscous or non-viscous, are thoroughly analyzed with respect to positivity and existence or non-existence of solutions, all depending on the constitutive relation for the pressure law. We distinguish between isothermal (linear) and isentropic (power law) pressure functions of the density. It is proven that in the dispersive, non-viscous model, a classical positive solution only exists for "small" (positive) particle current densities, both for the isentropic and isothermal case. Uniqueness is also shown in the isentropic subsonic case, when the pressure law is strictly convex. However, we prove that no weak isentropic solution can exist for "large" current densities. The dispersive, viscous problem admits a classical positive solution for all current densities, both for the isentropic and isothermal case, with an "hyper-diffusion" condition.

The proofs are based on a reformulation of the equations as a singular elliptic second-order problem and on a variant of the Stampacchia truncation technique. Some of the results are extended to general third-order equations in any space dimension.

Finally, the semi-classical and the inviscid limit are rigorously performed in the one-dimensional case. It is shown that the semi-classical and inviscid limit commute for sufficiently small data (i.e. current density) corresponding to subsonic states, were the inviscid non-dispersive solution is regular. In addition, we show these limits do *not* commute in general. The proofs are based on a reformulation of the problem as a singular second-order elliptic system and on elliptic and $W^{1,1}$ estimates.

Presented by Irene M. Gamba