On an Inequality by N. Trudinger and J. Moser

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It was shown by N. Trudinger and J. Moser that for functions in $H_0^1(\Omega)$ with $\int_\Omega |\nabla u|^2 dx \leq 1$ (where Ω is a bounded domain in $\mathbb{R}^2$) the integral $\int_\Omega \exp(\alpha u^2) dx$ remains uniformly bounded by some constant $C_\alpha = C_\alpha(\Omega)$, for $\alpha \leq 4\pi$, and becomes infinite for $\alpha > 4\pi$. L. Carleson and A. Chang proved that for the limting case $\alpha = 4\pi$ there exists a corresponding extremal function, in the case that Ω is the unit ball in $\mathbb{R}^2$. For proving this they introduced the maximal limit of all "concentrating" sequences of normalized functions; let us call this limit the "Carleson-Chang limit".

We give a new proof, a generalization, and a new interpretation of this result. In particular, we give an explicit sequence which converges to the Carleson-Chang limit.

The constant $C_{4\pi}(\Omega)$ mentioned above becomes infinite for unbounded domains. We show that there exists a uniform bound, independent of Ω, if the Dirichlet norm is replaced by the standard H_0^1-norm $\|u\|^2 = \int_\Omega |\nabla u|^2 + |u|^2 dx$. Furthermore, an explicit formula for the corresponding Carleson-Chang limit is given.