LEZIONE 14 DICEMBRE

10 GENNAIO 4 · 11 GENNAIO COMPITO

FINE GENNAIO 4

INIZIO FEBBRAIO 4

ESERCIZIO 69

$y(t)$ = quantità della sostanza radioattiva

$$\boxed{y'(t) = -\alpha\, y(t)}$$

t = tempo in ore

$y(t)$ = massa in grammi

$$\boxed{y(t) = C e^{-\alpha t}} = C\,(e^{\alpha})^{-t} = C\,\beta^{-t} \qquad \beta = e^{\alpha}$$

$$\boxed{y(t+24) = \frac{1}{2}\, y(t)}$$

la massa dimezza ogni 24 ore

$$C e^{-\alpha t - \alpha 24} = \frac{1}{2} C e^{-\alpha t}$$

$$\cancel{C} \cancel{e^{-\alpha t}} e^{-\alpha 24} = \frac{1}{2} \cancel{C} \cancel{e^{-\alpha t}} \qquad e^{-\alpha 24} = \frac{1}{2}$$

$$e^{\alpha 24} = 2$$

$$e^{\alpha} = \sqrt[24]{e^{\alpha 24}} = \sqrt[24]{2} = 2^{1/24} \qquad \alpha = \log_e 2^{1/24} = \frac{1}{24}\log_e 2$$

$$y(t) = C\,(e^{\alpha})^{-t} = C\, 2^{-\frac{t}{24}}$$

$t = 24$ ore $\quad y(t) = 100$ libbre $= 45'000$ gr.

$$C\, 2^{-\frac{24}{24}} = y(24) = 45'000$$

$$\frac{1}{2} C = 45'000 \qquad C = 90'000$$

$$\boxed{y(t) = 90'000\; 2^{-\frac{t}{24}}}$$

t esprime il tempo in ore

$y(s)$

s = tempo in secondi

$1\,h = 60\,m = 60 \cdot 60\,sec = 3600\,sec$ $\qquad t\,h = 3600\,t\ sec$

$s = 3600\,t$

$\boxed{t = \frac{s}{3600}}$

Sia $y(t)$ la funzione che esprime la massa al variare di t espresso in ore

$z(s)$ " " " " " " al variare di s espresso in sec.

$$z(s) = y\left(\frac{s}{3600}\right) = 30'000\, e^{-\frac{s}{3600 \cdot 25}}$$

$\boxed{70}$ $y(t)$ il n° di batteri al tempo t

$$\begin{cases} y' = y - y^2 \\ y(0) = 2. \end{cases}$$

- Calcolare $y(t)$
- Dire qualcosa sulla funzione

$y' = \alpha y - \beta y^2$

$y = \frac{\alpha/\beta}{1 - C e^{-\alpha t}}$

$$\frac{y'}{y - y^2} = 1$$

$$\int_0^t \boxed{\frac{y'}{y - y^2}}\, dt = \int_0^t 1\, dt = t$$

$f(z) = \frac{1}{z - z^2}$ $\qquad g(t) = y(t)$ $\qquad g'(t) \cdot f(g(t)) = \frac{y'}{y - y^2}$

$$\int_{y(0)}^{y(t)} \frac{1}{z - z^2}\, dz$$

$$\boxed{\int_{a=0}^{b=t} \underline{f(g(t))\ g'(t)}\, dt = \int_{g(a)=y(0)}^{g(b)=y(t)} f(s)\, ds}$$

$g(t) = y(t)$

$f(z) = \frac{1}{z - z^2}$

$$\frac{1}{z(1-z)} = \boxed{\frac{1}{z - z^2}} = \frac{A}{z} + \frac{B}{1-z} = \boxed{\frac{A - Az + Bz}{z(1-z)}}$$

$A = B$

$A = 1$

$$\frac{1}{z - z^2} = \frac{1}{z} + \frac{1}{1-z}$$

$$\int_2^{y(t)} \frac{1}{z} + \frac{1}{1-z} \, dz = \left[\log(z)\right]_2^{y(t)} - \int_2^{y(t)} \frac{1}{z-1} \, dz .$$

$$= \left[\log(z)\right]_2^{y(t)} - \left[\log(z-1)\right]_2^{y(t)}$$

$$t = \log(y(t)) - \log(y(t)-1) - \log 2 + \underbrace{\log 1}_{= 0}$$

$$\boxed{t = \log\left(\frac{y}{y-1}\right) - \log(2) = \log\left(\frac{y}{2(y-1)}\right)}$$

$$\boxed{\begin{aligned} &\log(a) - \log(b) = \log\left(\frac{a}{b}\right) \\ &\log(z)' = \frac{1}{z} \end{aligned}}$$

$$t = \log\left(\frac{y}{2(y-1)}\right)$$

$$e^t = \frac{y}{2(y-1)} \qquad \boxed{2e^t = \frac{y}{y-1}}$$

$$2e^t(y-1) = y$$

$$2e^t y - y = 2e^t$$

$$y(2e^t - 1) = 2e^t \qquad y = \frac{\cancel{2e^t} \,/\, \cancel{2e^t}}{(2e^t - 1) \,/\, 2e^t}$$

$$\boxed{y(t) = \frac{1}{1 - \frac{1}{2}e^{-t}}}$$

$$\lim_{t \to +\infty} \frac{1}{1 - \underbrace{\frac{1}{2}e^{-t}}_{\to 0}} = \frac{1}{1} = 1 \qquad = \frac{\alpha}{\beta}$$

VOGLIO CAPIRE SE $y(t)$ è crescente o decrescente.

VOGLIO CAPIRE IL SEGNO DI y'.

NATURALMENTE POSSO FARE IL CONTO.

$$y' = y - y^2 \qquad \underline{\underline{y'(0)}} = 2 - 4 = -2$$

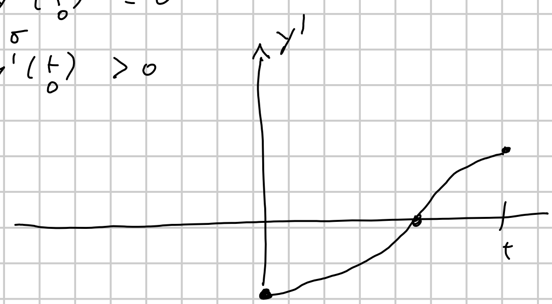
$$y(0) = 2$$

VOGLIO DIMOSTRARE CHE $y' < 0$ PER OGNI t.

SUPPONIAMO CHE NON SIA VERO

ESISTE t_0: $y'(t_0) \geq 0$.

$y(0) < 0$ $\qquad y'(t_0) = 0$ o $y'(t_0) > 0$



IN OGNI CASO ESISTE UN TEMPO t_1 TALE CHE $y'(t) = 0$

$$\begin{cases} y'(t) = y(t) - y(t)^2 \\ y(t_1) = B \end{cases} \qquad y'(t_1) = 0 = B - B^2 = 0 \qquad \boxed{\begin{array}{l} B = 1 \\ B = 0 \end{array}}$$

$$\boxed{\begin{cases} y'(t) = y(t) - y(t)^2 \\ y(t_1) = 1. \end{cases}} \qquad \boxed{y(t) = 1 \text{ PER OGNI } t}$$

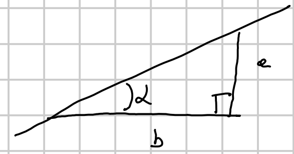
MA LA NOSTRA SOLUZIONE IN $t = 0$ VALE 2
QUINDI NON È QUESTA.

ESERCIZIO 71

$$\begin{cases} y' = 1 + y^2 \\ \underline{y(1)} = 1 \end{cases}$$

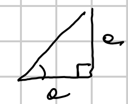
$$\int_1^t \frac{y'}{1 + y^2} = \int_1^t 1 = t - 1$$

$$\int_{y(1)}^{y(t)} \frac{1}{1+z^2}\,dz = \Big[\operatorname{arctg}(z)\Big]_1^{y(t)} = t-1$$



$$\operatorname{arctg} y(t) - \underbrace{\operatorname{arctg}(1)}_{= \frac{\pi}{4}} = t-1$$

$$tg(\alpha) = \frac{a}{b} = 1$$



$$\operatorname{arctg}(y) = t - 1 + \frac{\pi}{4}$$

$$\boxed{y(t) = tg\left(t - 1 + \frac{\pi}{4}\right)}$$

$\boxed{78}$

$$\underline{c(t) = c_0\, 3^{-2t}}$$

1) $t = 2$ $\qquad \boxed{c(2) = 2}$ $\qquad c_0\, \underbrace{3^{-2\cdot 2}} = 2$

$$c_0 = 2 \cdot 3^{2\cdot 2} = 2 \cdot 9^2 = 2\ 81 = 162$$

$$c(t) = 162\ 3^{-2t} \qquad c(0) = 162$$

2) $\boxed{c(t) = \frac{1}{2}\, c(0)} = \frac{1}{2}\, C_0 \qquad 3^{-2\cdot 0} = 1.$

$$\boxed{C_0\, 3^{-2t}} = \frac{1}{2}\, C_0\, 3^{-2\cdot 0} = \frac{1}{2}\, C_0$$

$$\frac{1}{2}\cancel{C_0} = \cancel{C_0}\, 3^{-2t}$$

$$\frac{1}{2} = \frac{1}{9^t}$$

$$\cancel{C_0}\, 3^{-2t} = \frac{1}{2}\cancel{C_0}$$

$$9^t = 2$$

$$\frac{1}{3^{2t}} = \frac{1}{2} \qquad \frac{1}{9^t} = \frac{1}{2}$$

$$t = \log_9 2$$

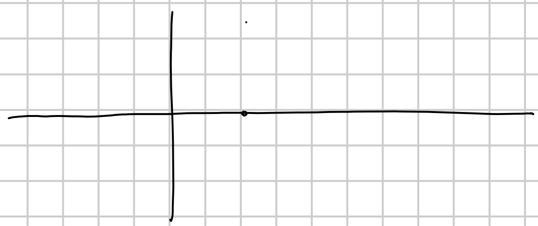
$$(3^2)^t \qquad 9^t = 2 \qquad t = \boxed{\log_9 2}$$

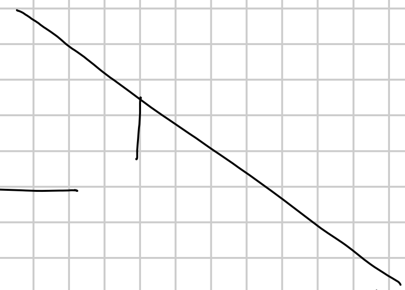
78 79 $\boxed{80}$ $\boxed{81}$

$\boxed{79}$

$f: (-3, +\infty) \to \mathbb{R}$ monotona decrescente

$(1, -5)$





$f(x) = -x + b$ $(1, -5)$

$f(1) = -5$

$$\begin{cases} f(x) = -x - 4 \\ f: (-3; +\infty) \to \mathbb{R} \end{cases}$$

$\log(x)$ $x > 0$

$f(x) = -\log_4(x+3) - 4$ $x > -3$

$f(1) = -\log_4(4) = -1.$

$$\boxed{f(x) = -\log_4(x+3) - 4}$$

$\sqrt{}$

$$\boxed{f(x) = -x - 4}$$

$f(x) = e^x$ log . .

$\boxed{\log(x)}$ $x > 0$

$(-3, +\infty) \to \mathbb{R}.$

$\boxed{-\log_4(x+3) - 4}$ $x + 3 > 0$

$\boxed{x > -3}$

-1

$\boxed{81}$

$I(t)$

$$\frac{\tan(t+1)}{2}\frac{dI}{dt} = I \qquad\qquad \frac{dI}{dt} = I' = DI = \dot{I}$$

$$\int_0^{x} \frac{I'}{I}\,dt = 2\int_0^{x}\frac{1}{\tan(t+1)}\,dt \qquad\qquad \frac{y'}{y-y^2} \qquad \int 1$$

$$\parallel$$

$$\int_{I(0)}^{I(x)} \frac{1}{z}\,dz = 2\int_0^{x}\frac{\cos(t+1)}{\sin(t+1)}\,dt = \int_{\sin(1)}^{\sin(x+1)}\frac{1}{z}\,dz$$

$$\Big[\log(z)\Big]_{I(0)}^{I(x)} = \Big[\log z\Big]_{\sin(1)}^{\sin(x+1)}$$

$$\log(I(x)) - C = \log(\sin(x+1)) - D$$

$$\log(I(x)) = \log(\sin(x+1)) + \underset{\substack{\uparrow\\ \text{una costante}}}{E}$$

$$I(x) = e^{E}\sin(x+1)$$

$$\boxed{I(x) = k\sin(x+1)}$$

- TROVA LA SOLUZIONE PARTICOLARE TALE CHE

$$I(1) = 3\sin(2)^2$$

$$I(1) = k\cancel{\sin(2)} = 3\sin(2)^{\cancel{2}}$$

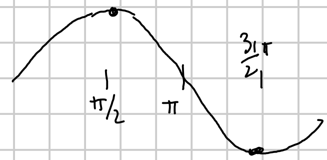
$$\boxed{k = 3\sin(2)}$$

$$I(x) = 3\sin(2)\,\boxed{\sin(x+1)}$$

$$\vee$$
$$0$$

$\sin(y)$ ha i massimi

in $y = \frac{\pi}{2} + 2k\pi \quad n\in\mathbb{Z}$



sin (y) ha i min.

$$y = \frac{3}{2}\pi + 2n\pi \qquad n \in \mathbb{Z}$$

$$I(x) = 3\sin(2)\ \underbrace{\sin(x+1)}$$

$$\boxed{\begin{aligned} x &= \frac{\pi}{2} - 1 + 2n\pi \\ x &= \frac{3\pi}{2} - 1 + 2n\pi \end{aligned}}$$

$$x + 1 = \frac{\pi}{2} + 2n\pi$$

$$x + 1 = \frac{3\pi}{2} + 2n\pi .$$