Magnetic Vortices, Nielsen-Olesen - Nambu strings and automorphic functions

Israel Michael Sigal

Department of Mathematics, Univ. of Toronto, Toronto, ON, Canada

Abstract.

The Ginzburg - Landau theory was first developed to explain and predict properties of superconductors, but had a profound influence on physics well beyond its original area. It had the first demonstration of the Higgs mechanism and it became a fundamental part of the standard model in the elementary particle physics. The theory is based on a pair of coupled nonlinear equations for a complex function (called order parameter or Higgs field) and a vector field (magnetic potential or gauge field). They are the simplest representatives of a large family of equations appearing in physics and mathematics. (The latest variant of these equations is the Seiberg - Witten equations.) Geometrically, these are equations for the connection on a principal bundle and the section of an associated vector bundle. Besides of importance in physics, they contain beautiful mathematics (some of the mathematics was discovered independently by A. Turing in his explanation of patterns of animal coats). In this talk I will review recent results involving key solutions of these equations - the magnetic vortices and vortex lattices, their existence, stability and dynamics, and how they relate to the modified theta functions appearing in number theory and algebraic geometry. Certain automorphic functions play key role in theory described.

.