Instability of solitary waves for nonlinear Schrödinger equations of derivative type

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Abstract.

We consider a nonlinear Schrödinger equation of derivative type:

$$i\partial_t u + \partial_x^2 u + i|u|^2\partial_x u + b|u|^4 u = 0, \quad (t,x) \in R \times R, \tag{1}$$

where $b \geq 0$. Eq (1) has a two parameter family of solitary wave solutions

$$u_\omega(t,x) = e^{i\omega_0 t}\phi_\omega(x - \omega_1 t),$$

where $\omega = (\omega_0, \omega_1) \in \Omega := \{(\omega_0, \omega_1) \in R^2 : \omega_1^2 < 4\omega_0\}$,

$$\phi_\omega(x) = \tilde{\phi}_\omega(x) \exp\left(i\frac{\omega_1}{2}x - \frac{i}{4}\int_{-\infty}^{x} |\tilde{\phi}_\omega(\eta)|^2 \, d\eta \right),$$

$$\tilde{\phi}_\omega(x) = \left\{ \frac{4\omega_0 - \omega_1^2}{-\frac{\omega_1}{2} + \sqrt{\omega_0 + \frac{4}{3}b(4\omega_0 - \omega_1^2)}\,\cosh(\sqrt{4\omega_0 - \omega_1^2}\,x)} \right\}^{1/2}.$$

The orbital stability of solitary waves $u_\omega(t)$ has been studied by Guo and Wu (1995) and Colin and Ohta (2006) for the case $b = 0$. In this talk, we consider the case $b > 0$, and prove the orbital instability of $u_\omega(t)$ for some ω.