Effective dynamics for the cubic nonlinear Schrodinger equation confined by domain or potential.

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Abstract.

We start by considering the cubic nonlinear Schrodinger (NLS) equation confined in a 2D box of size L. By taking the large box limit in a particular regime of small data, we derive a new equation that describes the effective dynamics of NLS over long nonlinear time scales. The obtained equation is called the "continuous resonant" (CR) equation and can also be interpreted as the equation for high frequency envelopes of NLS on the 2-torus. The (CR) equation turns out to satisfy rather surprising properties and symmetries, like leaving the Fourier transform and the eigenspaces of the quantum harmonic oscillator invariant by its flow. This signals to a relationship between the (CR) equation and the NLS equation on R^d with harmonic trapping. Indeed, we will see how the dynamics of (CR) can be used to describe the long-time behavior of NLS in both isotropic and non-isotropic (cigar-shaped) harmonic traps.

I will present works in collaboration with E. Faou, P. Germain, and L. Thomann.