







#### As much as possible Extreme value tasks in geometry

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# A typical task ...

We have a rectangular triangle with a = 3 cm, b = 4 cm, c = 5 cm, and we wantto inscribe the largest (area) rectangle when one side of it lays on *c*.





### ... with a typical answer?

#### First guess of most students:



A square ???



# How to find out the answer?

Calculus (Analysis): Area = e.f, and then ...

$$\frac{3}{4}f + e + \frac{4}{3}f = 5$$

From which we can easily derive a relation between *e* and *f* manely

$$e = 5 - \left(\frac{3}{4} + \frac{4}{3}\right)f = 5 - \frac{25}{12}f$$

Now we can finally write the equation for the function Area:

Area
$$(f) = e \cdot f = (5 - \frac{25}{12}f) \cdot f = 5f - \frac{25}{12}f^2$$

To get the maximum of this function, we first calculate the derivative:

$$Area'(f) = 5 - 2 \cdot \frac{25}{12} f = 5 - \frac{25}{6} f$$

Now we set Area'(f) = 0 and calculate the value for f:

Area'(f) = 
$$0 = 5 - \frac{25}{5}$$
 f  $\Rightarrow 5 = \frac{25}{5}$  f  $\Rightarrow f = \frac{6}{5} = 1.2$   
Let's stop here for a moment, because d a maximum at  $f = 1.2$ . To most students stop to think here, too ... e at  $f = 1.2$ :  
 $Area''(f) = -\frac{25}{6} < 0$ 



# Finding out an approximate answer with GeoGebra

- Make the construction in GeoGebra
- Calculate Area = e-f
- We (again) want to find out for which value of *f* we have the maximum value of *Area*
- Construct a point *P* with coordinates
  *P* = (*f*, *Area*)
- Try out different rectangles
- Use button "trace" (stopa zapnutá) to draw the graph



## Is "a square" correct?



Apparently not!



#### So what is the answer?



*f* ≈ 1.2 cm, *e* ≈ 2.5 cm



# And now, if you insist ...

... you can ask "why the analytic solution might be interesting?"

- GeoGebra gives you only an approximation, and you might want to know the exact solution
- This is no proof! (some might disagree)
- It is useful to know a general analytical approach, as the GeoGebra method has limitations



#### More of the same ...

#### Find the maximum area ...





... or the maximum volume:





# Conclusion

Don't hit everything on the head with calculus and derivatives right away ...

... but don't forget how to use calculus!

THANK YOU FOR YOUR ATTENTION! ĎAKUJEM ZA POZORNOSŤ!