Math Models in Biomedicine

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The team preparing the ebook is not homogeneous and it is composed by specialists in didactics of Mathematics as well as pure mathematicians without long experience in didactics. This particular point seems to become very useful since real interaction between different fields can give very positive impact on the whole work(but can provoke also negative results also!).

There were no particular restrictive schemes imposed at the beginning of the work.
1) The arguments in the didactic units had as an initial point models and events from the surrounding world
2)The arguments in the units have reasonable math contents supporting the preparation of future and in service teachers in Mathematics
3) We tried to unify all different arguments by using creative and attractive new ideas in posing, solving math problems and teaching math

The e-book "Dynamat" can be found on the homepage
http://www.dynamathmat.eu/
of the "Dynamat" Comenius project.

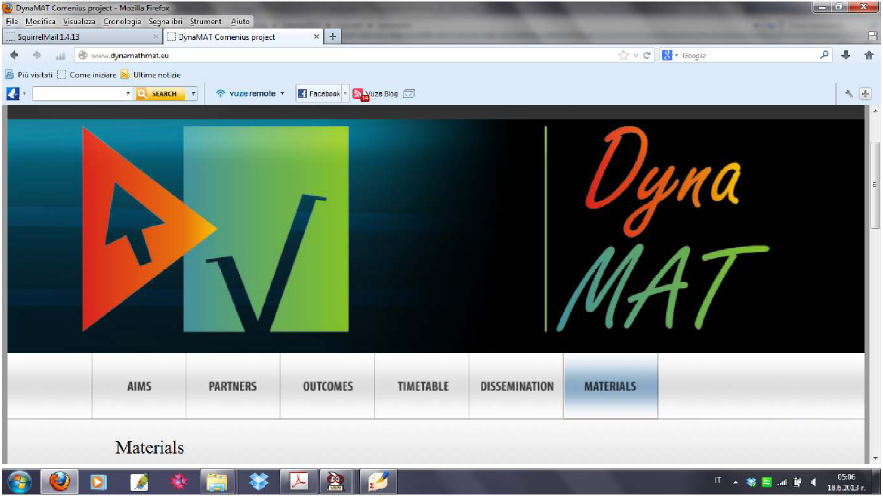


Figure: Dynamat

Some of the simplest recurrence sequences might be solved explicitly , for example

$$a_{n+1} = 2a_n, \quad a_1 = 1.$$

The corresponding graphics is in the next picture

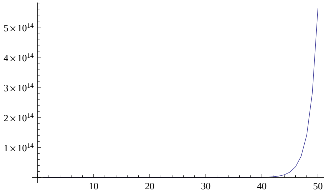


Figure: Simple recurrence relation

Some other famous recurrence sequences as Fibonacci one might be solved explicitly,

$$a_{n+1} = a_n + a_{n-1}, \quad a_1 = 1, a_2 = 1.$$

The corresponding graphics is in the next picture

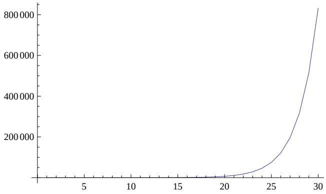


Figure: Simple recurrence relation

More complicated is the scalar DNLS (discrete nonlinear Schrödinger equation)

$$a_{n+1} = 2a_n - a_{n-1} - a_{n-1}^3, \quad a_1 = \alpha, a_2 = \beta.$$

There is no explicit formula for general initial data α, β. For example one can pose the question to find a couple (α, β) so that x_n is a decreasing, bounded sequence of positive numbers.

One can choose for example

$$\alpha = 0, \beta = 0.1$$

and compute (even with Excell) the first 15 terms for example. The corresponding graphics is as follows

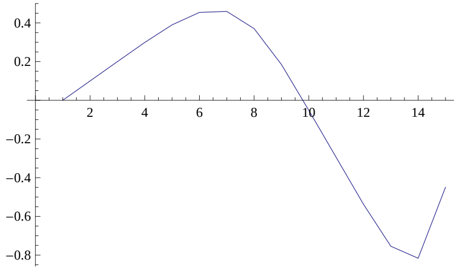
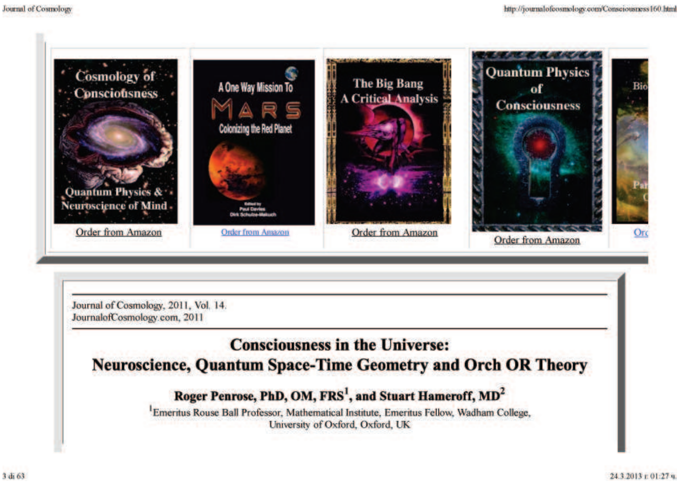


Figure: DNLS



Journal of Cosmology, 2011, Vol. 14.
JournalofCosmology.com, 2011

Consciousness in the Universe:
Neuroscience, Quantum Space-Time Geometry and Orch OR Theory

Roger Penrose, PhD, OM, FRS[1], and Stuart Hameroff, MD[2]
[1]Emeritus Rouse Ball Professor, Mathematical Institute, Emeritus Fellow, Wadham College, University of Oxford, Oxford, UK

Figure: Penrose article

NATURE|Vol 440|30 March 2006

ESSAY

Quantum mechanics in the brain

Does the enormous computing power of neurons mean consciousness can be explained within a purely neurobiological framework, or is there scope for quantum computation in the brain?

Christof Koch and Klaus Hepp

A thought experiment involving an observer looking at a superimposed quantum system with one eye, and at a succession of faces with the other, challenges the idea that a quantum framework is needed to explain consciousness.

The relation between quantum mechanics and higher brain functions, including consciousness, is often discussed, but is far from being understood. Physicists, ignorant of modern neurobiology, are tempted to assume a formal or even dualistic view of the mind–brain problem. Meanwhile, cognitive neuroscientists and neurobiologists consider the quantum world to be irrelevant to their concerns and therefore do not attempt to understand its concepts. What can we confidently state about the current relationship between these two fields of scientific inquiry?

All biological organisms must obey the laws of physics, both classical and quantum. In contrast to classical physics, quantum mechanics is fundamentally indeterministic. It explains a range of phenomena that

intact organisms. Most quantum physicists view the brain as a classical instrument.

The critical question we are concerned with here is whether any components of the

computation seeks to exploit the parallelism inherent in entanglement by assuring that the system is very likely to converge on the computa-

Figure: Neuroscience and quantum mechanics

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Research Article

Mozart K.545 Mimics Mozart K.448 in Reducing Epileptiform Discharges in Epileptic Children

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Figure: Mozart effect

Talk of Prof. Alfonso Iudice
EPILESSIA o EPILESSIE?

List of intelectuals suffuring from this desease:
Maometto, Socrate, Platone, Giulio Cesare, Petrarca, Carlo V, San Paolo, Blaise Pascal, Niccoló Paganini, George Byron, Vittorio Alfieri, Fedor Dostoevskij, Alessandro Magno, Giulio Cesare, Napoleone Bonaparte, George Frederick Handel, Nietzsche, Moliére, Flaubert, Torquato Tasso, Dickens, Francesco Petrarca, Lewis Carroll, il matematico Isaac Newton, il Cardinale Richelieu, papa Pio IX, Alfredo Nobel, Michelangelo Merisi detto il Caravaggio e Vincent Van Gogh (?).

Figure: Creation of Adam, Michelangelo and Neurons (Iudice)

NeuroQuantology | December 2010 | Vol 8 | Issue 4 | Page 564-570
Kozlowski and Marciak-Kozlowska. Schrödinger equation and human neuron mass 564

Original Article

Modified Schrödinger Equation for Particles with Mass of the Order of Human Neuron Mass

Miroslaw Kozlowski*, Janina Marciak-Kozlowska[†]

Abstract

In this paper the modified Schrödinger equation (MSE) for the particles with mass = mass of the human neuron is obtained and solved. Considering that neuron mass is of the order of Planck mass it was shown that for mass of the order of the human neuron mass the transition quantum → classical behavior can occurs. Moreover it was argued that the human brain can be described as the fluid of the Planck particles. It is interesting to observe that the Planck gas was created at the beginning of the Universe.

Key Words: Modified Schrödinger Equation, Planck particles, neurons.

NeuroQuantology 2010; 4: 564-570

Figure: Modified NLS

Stability of nonlinear first-order recurrences

$$x_n = f(x_{n-1}).$$

This recurrence is locally stable, meaning that it converges to a fixed point x^* from points sufficiently close to $x*$, if the slope of f in the neighborhood of $x*$ is smaller than unity in absolute value: that is,

$$|f'(x^*)| < 1. \tag{1}$$

A nonlinear recurrence could have multiple fixed points, in which case some fixed points may be locally stable and others locally unstable.

Simplified version of the discrete nonlinear Schrödinger equation

$$a_{n+1} = a_n - a_{n-1}^2, \quad a_1 = \alpha.$$

There is explicit formula for initial data $\alpha = 1$, i.e. $a_n = 0$ for $n \geq 2$. If $\alpha = 0.5$ we have

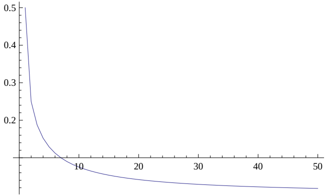

Figure: Simplified first order recurrence sequence

One can verify the following assertions

1. for any $\alpha \in (0,1)$ the recurrence sequence

$$a_{n+1} = a_n - a_{n-1}^2, \quad a_1 = \alpha$$

   is a decreasing sequence of positive numbers
2. the sequence a_n tends to 0.

Consider a two-species predator-prey discrete model in which one species preys on another. Examples in the nature include sharks and fish, lynx and snow-shoe hares, ladybirds and aphids, wolves and rabbits. A very simple model, known as the Lotka-Volterra model is the following:

$$\begin{aligned} x_{n+1} &= x_n(1 + p_1 - p_2 x_n - p_3 y_n) \\ y_{n+1} &= y_n(1 - q_1 + q_2 x_n), \quad n = 0, 1, 2, \ldots \end{aligned} \tag{2}$$

Here p_1, p_2, p_3, q_1 and q_2 are nonnegative constants; x_n and y_n represent the number of prey and predator populations respectively at time n. The terms appearing in the right-hand sides of the equations, have a biological meaning as follows:

• $(1+p_1)x_n - p_2x_n^2$ represents the logistic growth of the population of prey in the absence of predator;
• $p_3x_ny_n$ and $q_2x_ny_n$ represent species interaction: the population of prey suffers and predators gain from the interaction;
• $(1-q_1)y_n$ represents the extinction of predators in the absence of preys.

There are three particular types of outcome that are often observed in the real world. In the first case, there is coexistence, in which the two species live in harmony. In nature, this is the most likely outcome. In the second case, one of the species becomes extinct, and in the third case both species go to extinction.

Having some values at the initial time $n = 0$, say (x_0, y_0), we can consecutively compute by means of (2) an infinite sequence of points in the (x, y)-plane

$$(x_1, y_1),\ (x_2, y_2), \ldots, (x_n, y_n),\ (x_{n+1},\ y_{n+1}), \ldots \tag{3}$$

This sequence describes the evolution of the populations as time increases and is called *a trajectory of* (x_0, y_0); (x_0, y_0) is called *initial point* or initial condition. Obviously, the values of the sequence members depend on the choice of (x_0, y_0) as well as on the values of the constants p_1, p_2, p_3, q_1 and q_2. The main question is: given some initial point (x_0, y_0) what can we say about the behavior of the trajectory (3) for sufficiently large n? Figure 1(a) presents three trajectories within

$$p_1 = 0.05, \quad p_2 = 0.0001, \quad p_3 = 0.001, \quad q_1 = 0.03, \quad q_2 = 0.0002 \tag{4}$$

for three different initial conditions $(x_0, y_0) = (20, 5)$, $(x_0, y_0) = (100, 10)$ and $(x_0, y_0) = (50, 40)$, denoted by solid boxes. As you can see, when n increases, the three trajectories approach one point in the plane and remain close to it.

Such a point is called a *stable steady state* or *attractor*. You can find it by replacing in (2) $x_{n+1} = x_n = x$, $y_{n+1} = y_n = y$ and then solve the obtained nonlinear system for x and y. You will obtain three different solutions $(x, y) = (0, 0)$, $(x, y) = (500, 0)$ and $(x, y) = (150, 35)$. The third point $(150, 35)$ is the attractor, shown in Figure Lotka Volterra - variant A. The other two steady states are called *unstable*, because the trajectories of initial points, even slightly different from these steady states, move away from them as time n increases.

Now set $p_2 = 0$ in the model (2); in this way you change the growth rate law in the prey population. The system becomes

$$\begin{aligned} x_{n+1} &= x_n(1 + p_1 - p_3 y_n) \\ y_{n+1} &= y_n(1 - q_1 + q_2 x_n), \quad n = 0, 1, 2, \ldots \end{aligned} \tag{5}$$

Figure Lotka Volterra - variant B presents one trajectory with the same coefficient values for p_1, p_3, q_1 and q_2 from (4) and with initial condition $(x_0, y_0) = (20, 5)$, denoted by a solid box. Now we see a totally different picture: the two populations oscillate, building a *stable cycle*. How can the system (5) be interpreted in terms of species behavior? If the ratio of predators to prey is relatively high, then the population of predators drops. When the ratio of predators to prey drops, then the population of prey increases. If there is sufficient quantity of prey, the predator number starts to increase. The resulting cyclic behavior is repeated over and over.

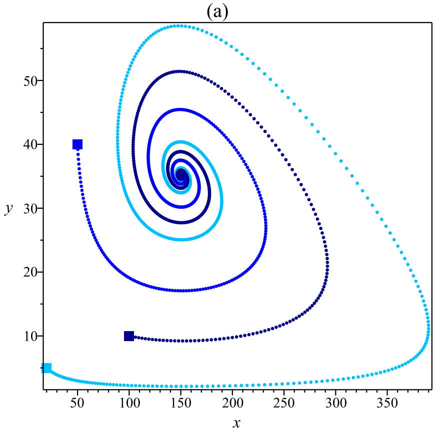

Figure: Lotka Volterra - variant A

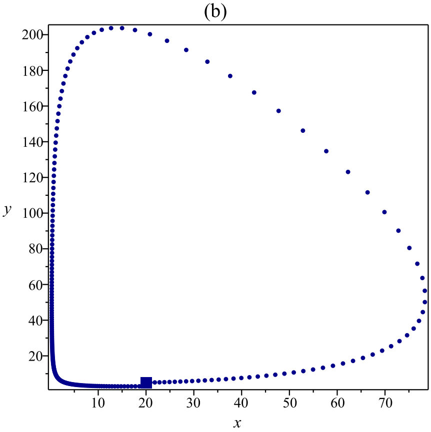

Figure: Lotka Volterra - variant B