Dynamical and creative Mathematics using ICT (DynaMAT)

Problem posing Lab and (mathematical or real life) input from teachers, students, teacher trainers etc.

1 Starting from a known problem using ICT make a conjecture.

1.1 Problem 1

A student of the famous Galileo Galilei discovered a new planet, which orbits the Sun on an elliptic orbit with semi-axes *a* and *b*. Suppose that an observer is located at a point in the plane of the ellipse, such that the ellipse is seen from that point at an angle of $90°$. Compute the distance between the center of the ellipse and the observation point.

The application used is based on Power Point application.

1.2 Solution to problem 1 (team pf Brescia, Italy)

We use Cartesian Coordinate System. The semi axes of the ellipse lie on the Cartesian axes, the centre of the ellipse is in the origin. The equation of the ellipse is:

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, let $P\left(x_0; y_0\right)$ be the point where the observer is, from where we draw the two tangent lines. Now we are looking for the angular coefficient of the tangent lines using the following system:

$$\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \\ y-y_0=m\left(x-x_0\right) \end{cases}$$

where the tangency coefficient is:

$$m_{1/2}=\frac{-x_0y_0\pm\sqrt{x_0{}^2y_0{}^2-\left(a^2-x_0{}^2\right)}\left(b^2-y_0{}^2\right)}{a^2-x_0{}^2}.$$

The tangent lines from the point must be perpendicular so for their two angular coefficients is valid the relation: $m_1\times m_2=-1$.

From where

$$\frac{-x_0y_0+\sqrt{x_0{}^2y_0{}^2-\left(a^2-x_0{}^2\right)}\left(b^2-y_0{}^2\right)}{a^2-x_0{}^2}\times\frac{-x_0y_0-\sqrt{x_0{}^2y_0{}^2-\left(a^2-x_0{}^2\right)}\left(b^2-y_0{}^2\right)}{a^2-x_0{}^2}=-1,$$

so we get:

$$x_0{}^2+y_0{}^2=b^2+a^2.$$

The last equation is the locus of points in which the observer can be and for which is true that the tangent lines of the ellipse are perpendicular. This locus is the circle wit center in the origin and radius $r=\sqrt{a^2+b^2}$ .The distance between the observer and the center of the ellipse is the same as the distance between a point on the circle and the center hence this distance is the radius and is $\sqrt{a^2+b^2}$.

1.3 Problem for the bus in Pisa

Given three fixed points A,B, C on the segment AC (the door of the bus that can be found in Pisa) on the axis Ox moving to the axis Oy so that

A=O ➔ A'

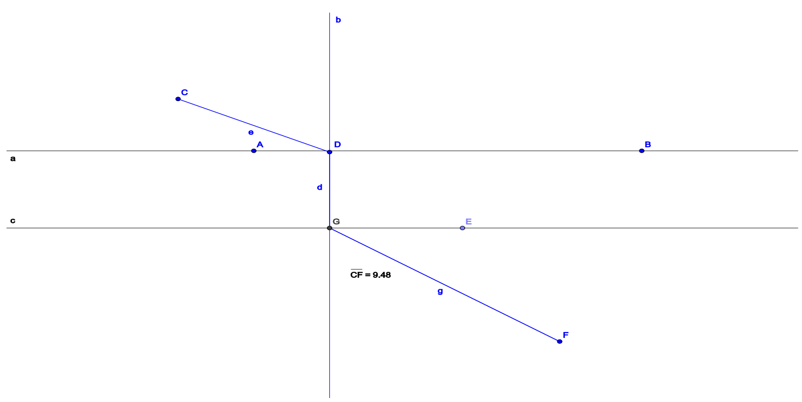
C ➔ O

The problem is to describe the movement of the fixed point B and define the curve of the movement of B.

The application used is based on Power Point application and enables one to conjecture that the curve is ellipse.

1.4 River problem

Given a river (two parallel lines a and c with a fixed distance d between them) and given two points C and F on the opposite sides of the river, find the shortest path connecting C and F and crossing the river on segment DG perpendicular to a and c.

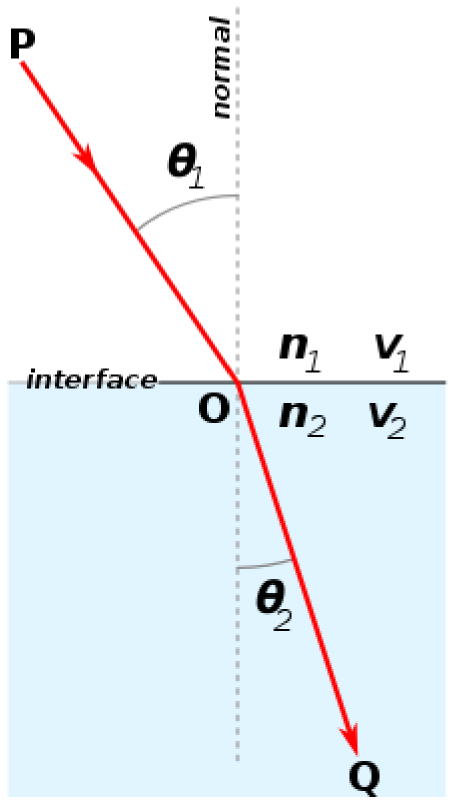


The dynamic application is done with geogebra.

Hint. One can arrive at the conjecture that the shortest path is defined, when CD and GF are parallel.

1.4 Snell's law and shortest path problem.

In optics and physics, **Snell's law** (also known as **Descartes' law**, the **Snell–Descartes law**, and the **law of refraction**) is a formula used to describe the relationship between the angles of incidence and refraction, when referring to light or other waves passing through a boundary between two different isotropic media, such as water and glass.



Refraction of light at the interface between two media of different refractive indices, with $n_2 > n_1$. Since the velocity is lower in the second medium ($v_2 < v_1$), the angle of refraction θ_2 is less than the angle of incidence θ_1; that is, the ray in the higher-index medium is closer to the normal.

In optics, the law is used in ray tracing to compute the angles of incidence or refraction, and in experimental optics and gemology to find the refractive index of a material. The law is also satisfied in metamaterials, which allow light to be bent "backward" at a negative angle of refraction (negative refractive index).

Although named after Dutch astronomer Willebrord Snellius (1580–1626), the law was first accurately described by the Persian scientist Ibn Sahl at Baghdad court, when in 984 he used the law to derive lens shapes that focus light with no geometric aberrations in the manuscript *On Burning Mirrors and Lenses* (984).[1][2]

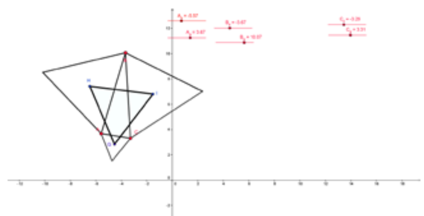
Snell's law states that the ratio of the sines of the angles of incidence and refraction is equivalent to the ratio of phase velocities in the two media, or equivalent to the opposite ratio of the indices of refraction:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

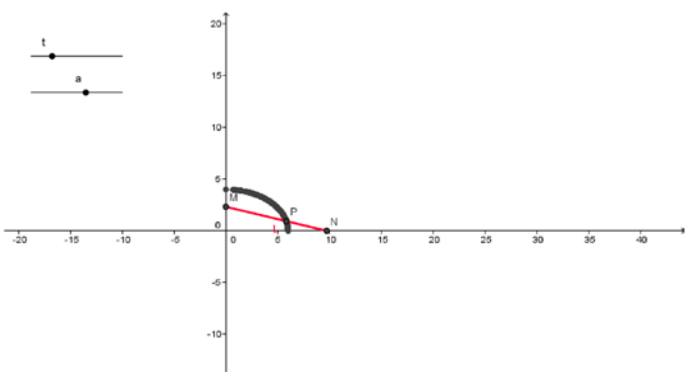
with each θ as the angle measured from the normal, v as the velocity of light in the respective medium (SI units are meters per second, or m/s) and n as the refractive index (which is unitless) of the respective medium.

The law follows from [Fermat]'s [principle of least time], which in turn follows from the propagation of light as waves.

Possible materials to be involved: Napoleon’s Theorem,



some materials from the collaboration with prof. Yuki Kurokawa: problems from samurai period(bus problem),



topics from dynamical systems and billiards: periodic triangles,