The problem explored by Anton and Kaloyan

Given a triangle ABC and a point M in its plane, let M_a, M_b and M_c be the reflections of M with respect to the angle bisectors at A, B, and C respectively. Then AM_a, BM_b and CM_c are the isogonal conjugate lines of AM, BM and CM respectively. They concur at a point N, called the **isogonal conjugate** of the point M.

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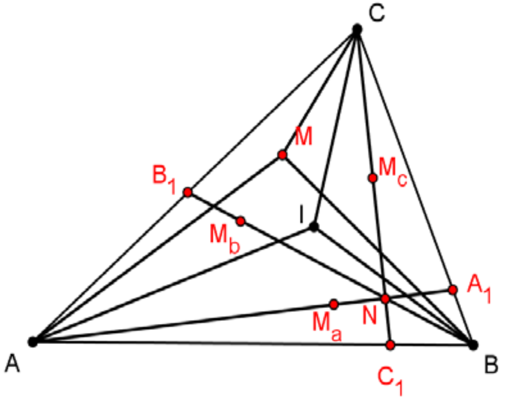

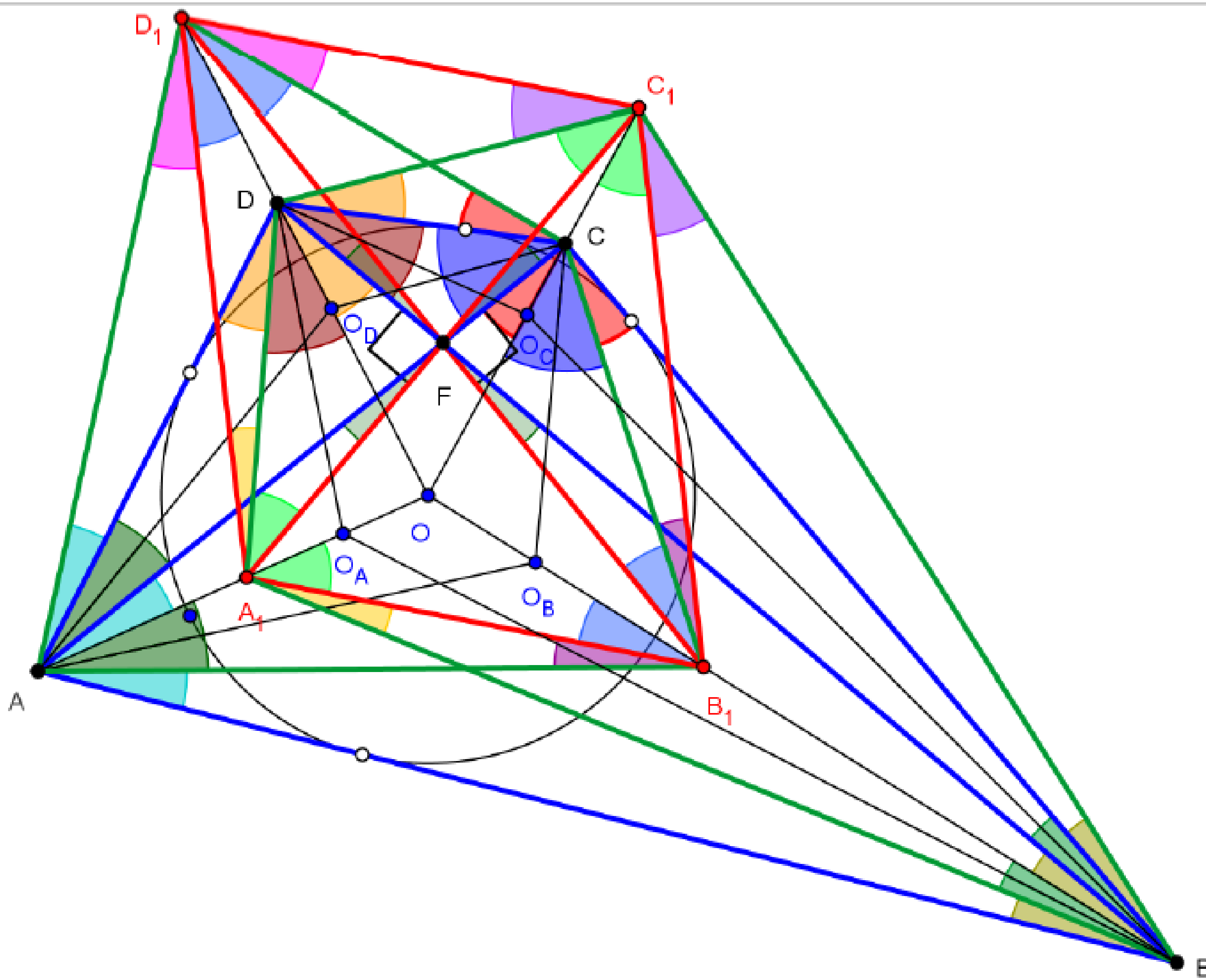

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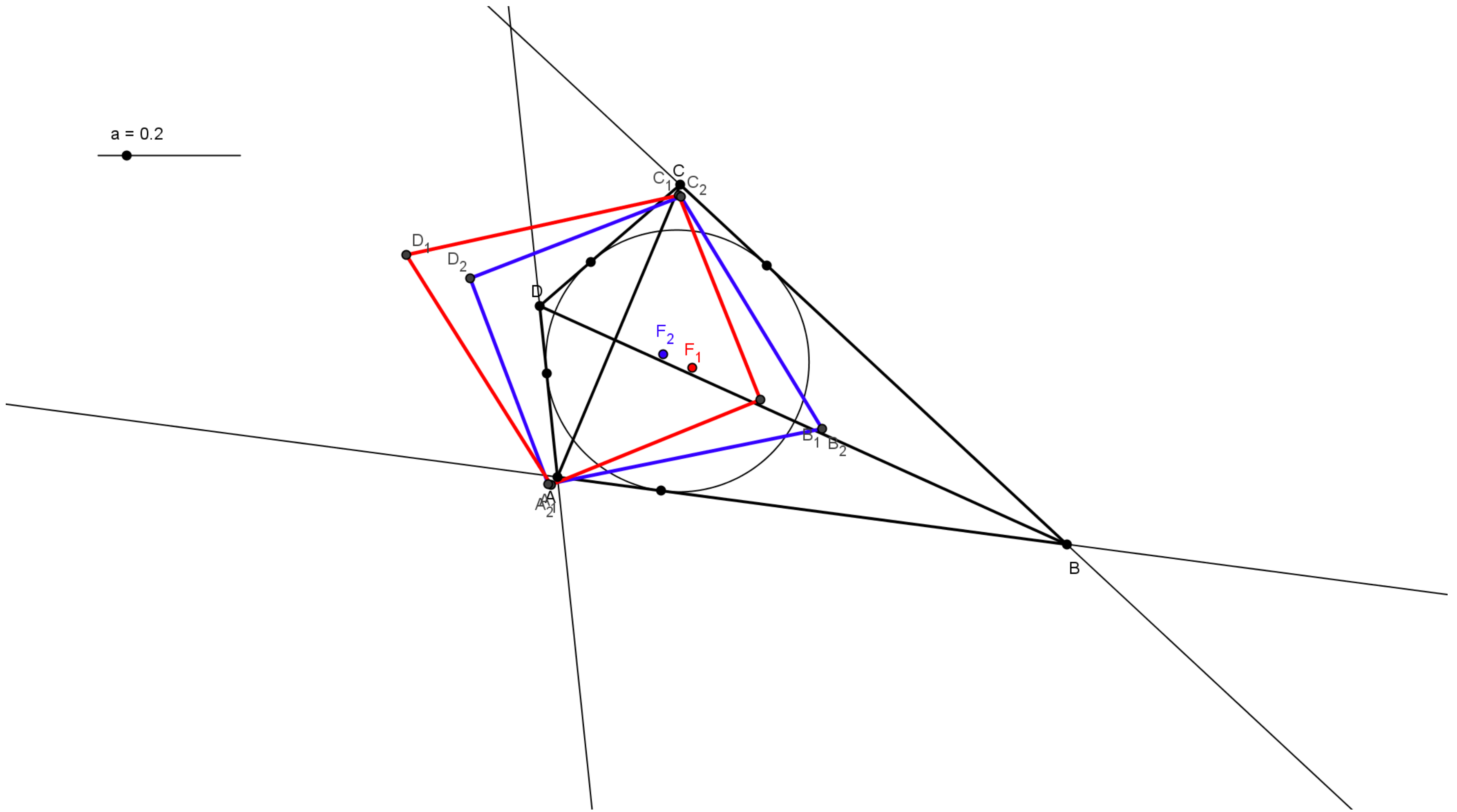
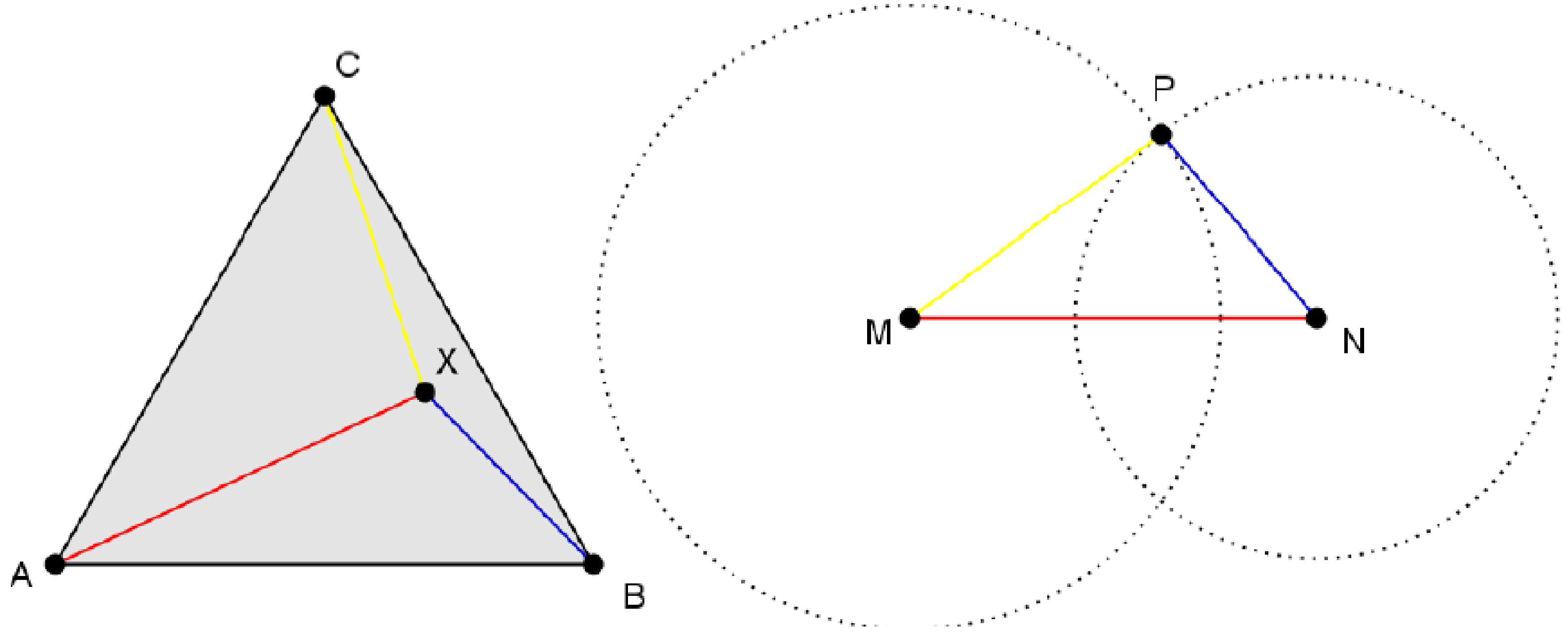
Exploring partial cases



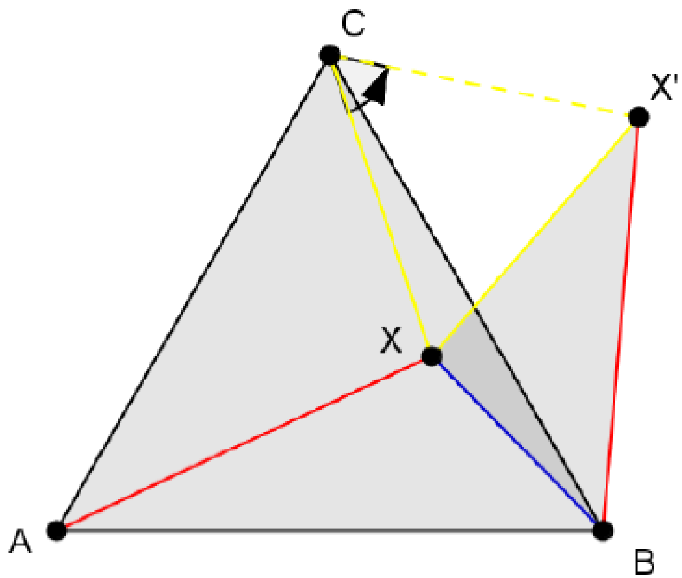
The problem attacked by Yanica Pehova

Using the obvious approach, one simply takes the distances from the point X to the vertices of the triangle and constructs a new triangle given three sides. This is what the construction looks like:



The tricky approach

Using the "tricky" approach though, one constructs Pempeiu's triangle as part of the initial configuration of the triangle ABC and point X, rather than separately, on the side, as we did above:

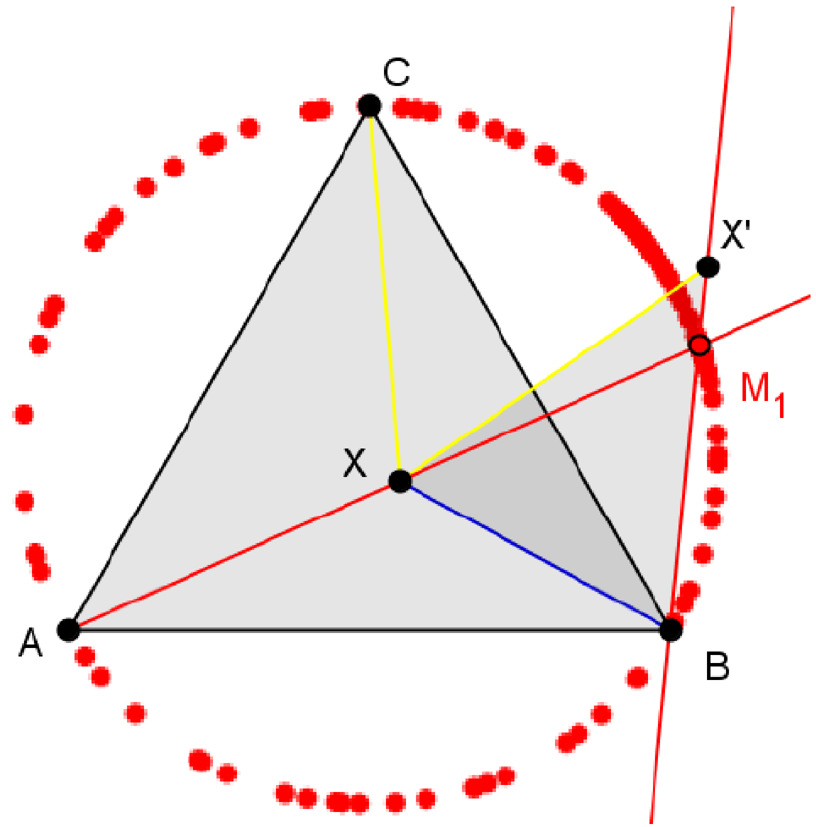


The point X' is the image of the point X under rotation of 60∘ about one of the vertices of the triangle ΔABC.

The sides of the triangle BXX' have lengths equal to the lengths of AX, BX and CX.

Hypotheses becoming theorems thanks to dynamic explorations

Conjecture 1: Let ΔXCX' be Pompeiu's triangle for ΔABC and the point X. Then, the intersection point of the ray AX→ and the line BX' lies on the circumscribed circle of ΔABC .

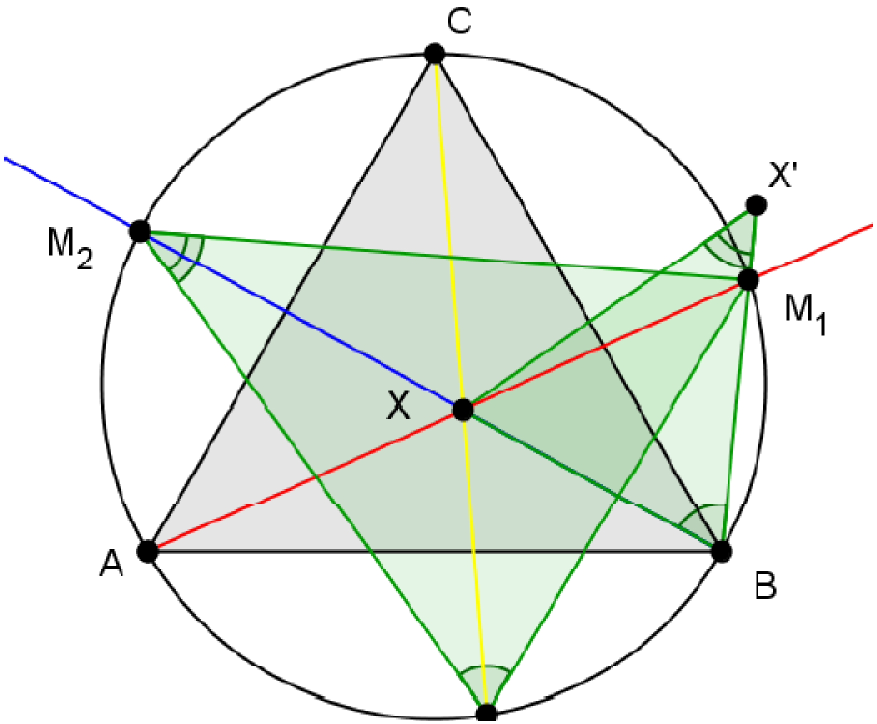


I could easily verify this, once I let GeoGebra trace out the intersection point (M1). Thus, the circumscribed circle ΔABC was formed.

It is worth mentioning that apart from A, B, C and M1, the points X, M1, X' and C also lie on a circle.

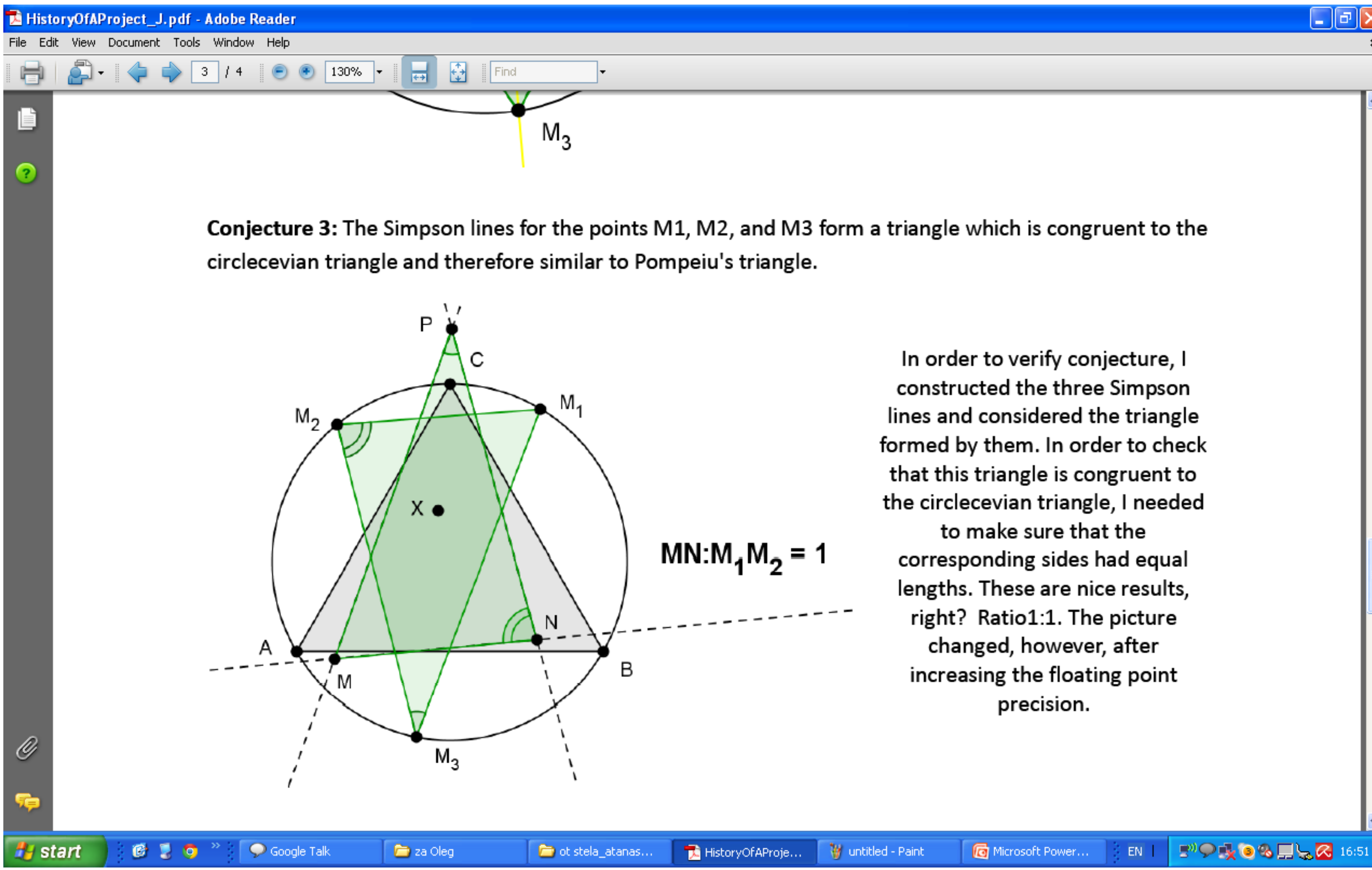
Hypotheses becoming theorems thanks to dynamic explorations

Conjecture 2: If in an analogous way we add the points of intersection, M2 and M3, of the rays BX→ and CX→ with the circumscribed circle of ΔABC then ΔM1M2M3 is similar to Pombeiu's triangle. (The triangle ΔM1M2M3 constructed in this way is called *circlecevian*)

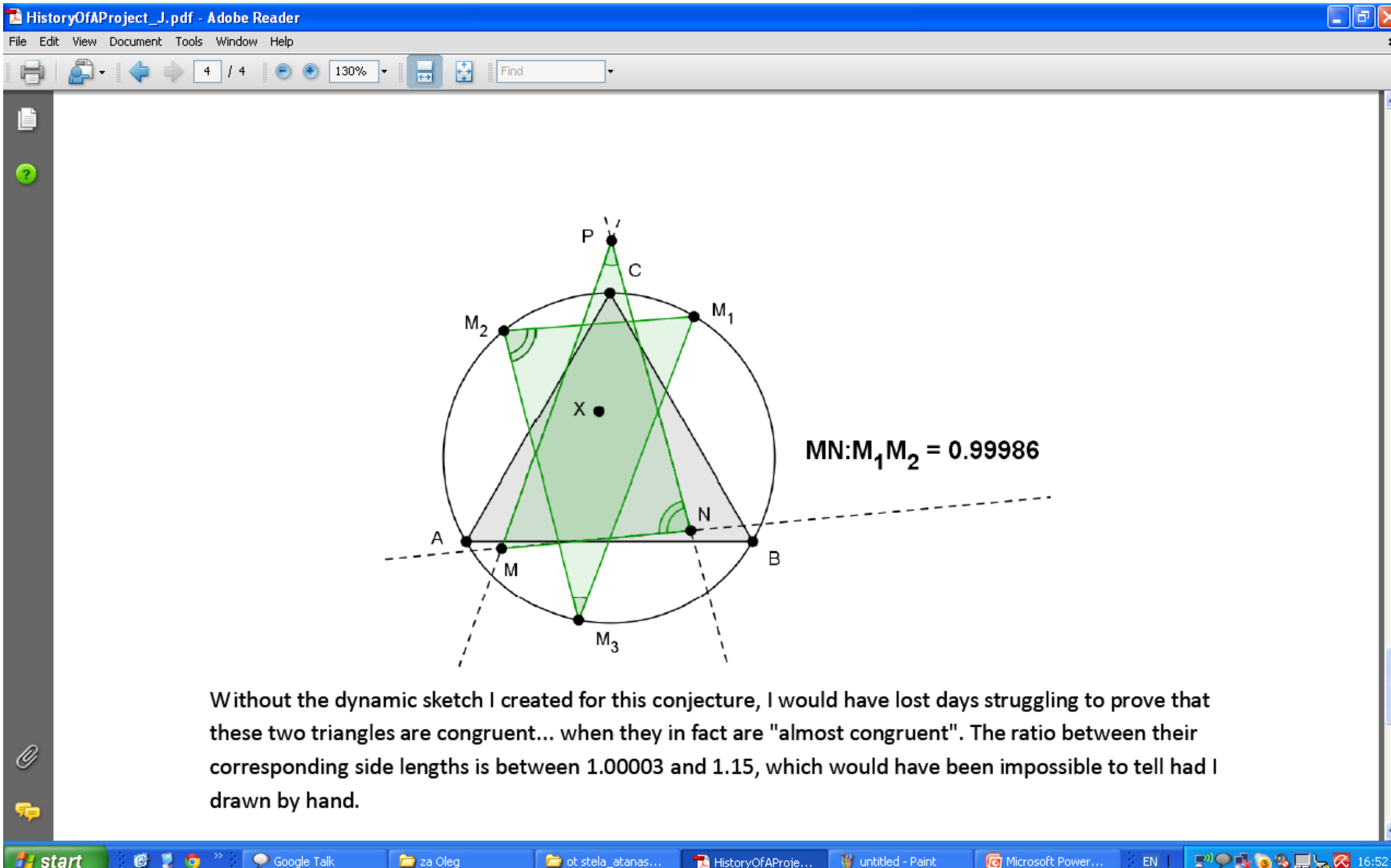


To show the similarity of the triangles, it suffices to show that two of the angles of ΔM1M2M3 are equal to two of the angles of Pompeiu's triangle. With this, GeoGebra's job is done, since for the proof one cannot rely on any software.

Hypotheses becoming theorems thanks to dynamic explorations

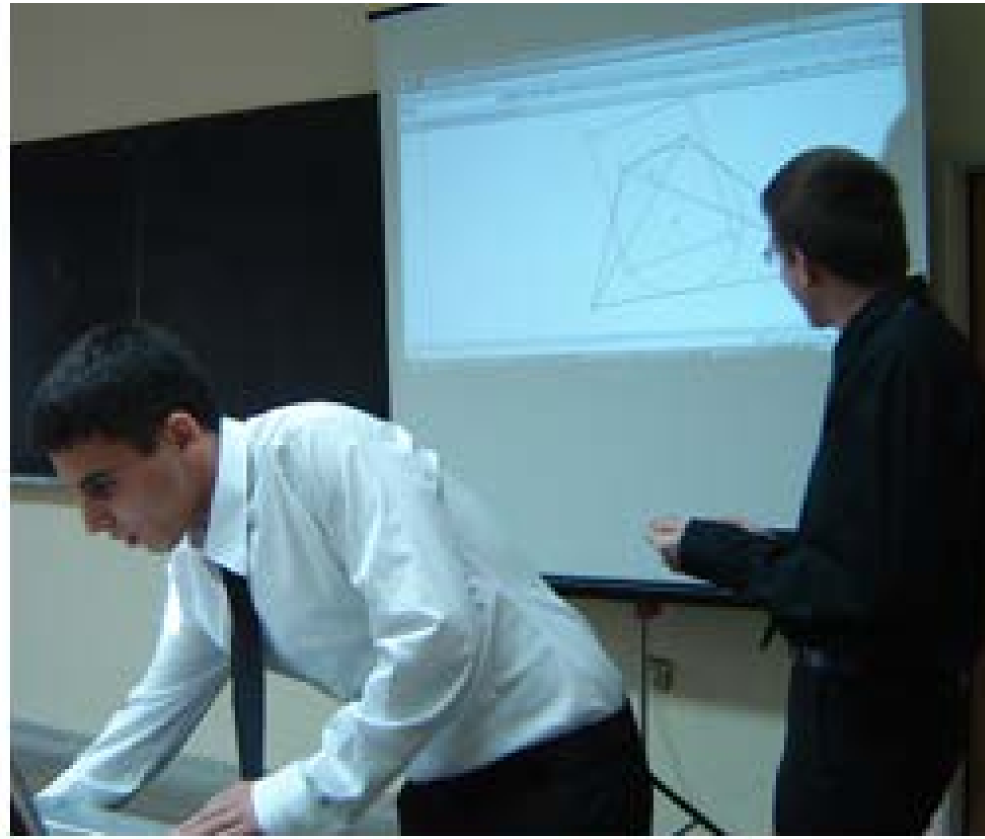


The dynamic sketch in help!



Without the dynamic sketch I created for this conjecture, I would have lost days struggling to prove that these two triangles are congruent... when they in fact are "almost congruent". The ratio between their corresponding side lengths is between 1.00003 and 1.15, which would have been impossible to tell had I drawn by hand.

That's what makes us happy!



The students