Configuration Spaces

Geometry, Topology and Representation Theory

Cortona, Italy

August 31 – September 6, 2014

Schedule

Monday, September 1

09:00–09:30	Registration
09:30–10:30	**Alexandru Suciu** (Northeastern University): *Topology of line arrangements*
10:30–11:00	Coffee break
11:15–12:15	**Michèle Vergne** (University of Paris 7): *Euler MacLaurin formula for rational polytopes*
12:30–14:00	Lunch
15:00–16:00	**Dmitry Kozlov** (University of Bremen): *Configuration spaces arising in distributed computing*
16:00–16:30	Coffee break
16:30–17:30	**Michael Farber** (University of Warwick) *Large random spaces and groups*

Tuesday, September 2

09:30–10:30   **Mike Davis** (Ohio State University):
                         Complements of hyperplane arrangements as posets of spaces

10:30–11:00   Coffee break

11:15–12:15   **Jon McCammond** (University of California, Santa Barbara):
                         The structure of Euclidean Artin groups

12:30–14:00   Lunch

15:00–16:00   **Emanuele Delucchi** (University of Fribourg):
                         Recent developments in toric arrangements

16:00–16:30   Coffee break

16:30–17:30   **Luca Moci** (University of Paris 7)
                         Matroids over a ring: motivations, examples, perspectives

17:45–19:15   **Poster session**

Wednesday, September 3

09:00–10:00   **Hal Schenck** (University of Illinois):
                         Chen ranks and resonance

10:00–10:30   Coffee break

10:30–11:30   **Graham Denham** (University of Western Ontario):
                         Elliptic braid groups are duality groups

11:45–12:45   **Mike Falk** (Northern Arizona University):
                         BGG resolutions and configuration spaces

13:00–14:00   Lunch

14:00–20:00   *Free afternoon and guided tour of Cortona*

20:00   **Conference Dinner**

Thursday, September 4

09:30–10:30   **Eric Babson** (University of California, Davis):
Random triangulations of the two sphere

10:30–11:00   Coffee break

11:15–11:45   **Anders Björner** (KTH Stockholm):
Filtered geometric lattices and tropical Lefschetz theorem, I

11:45–12:15   **Karim Adiprasito** (IHES, Paris):
Filtered geometric lattices and tropical Lefschetz theorem, II

12:30–14:00   Lunch

15:00–16:00   **Thomas Brady** (Dublin City University):
Non-crossing partitions and monodromy of Milnor fibres

16:00–16:30   Coffee break

16:30–17:30   **Matthias Franz** (University of Western Ontario)
Big polygon spaces

Friday, September 5

09:30–10:30   **Alejandro Adem** (University of British Colunbia):
An Infinite Loop Space Associated to Commuting Matrices

10:30–11:00   Coffee break

11:15–12:15   **Sergey Yuzvinsky** (University of Oregon):
Higher topological complexity of Eilenberg-MacLane spaces

12:30–14:00   Lunch

15:00–16:00   **Gus Lehrer** (University of Sydney):
Cohomology of the Milnor fibre of an arrangement

16:00–16:30   Coffee break

16:30–17:30   **Claudio Procesi** (Università di Roma I)
From the theorem of Amitsur-Levitzki to exterior algebras of simple Lie algebras

Titles and abstracts of talks

Alejandro Adem (University of British Columbia)

Title: An Infinite Loop Space Associated to Commuting Matrices.

Abstract: Let G denote a Lie group. We show that a construction introduced by Adem-Cohen-Torres built out of the commuting elements in G plays the role of a classifying space for commutativity. We will discuss how this is reflected in properties of these spaces and show that for the unitary group U we obtain a new infinite loop space. This leads to the notion of commutative K-theory, with characteristic classes computed using multisymmetric polynomials. This is joint work with José Gómez, John Lind and Ulrike Tillmann.

Eric Babson (University of California, Davis)

Title: Random triangulations of the two sphere.

Abstract: We study the local geometry of random triangulations of the two sphere by constructing a foliated configuration space with a measure and a metric so that the metric balls in its leaves are those in the triangulations with the measure recording their frequency. This construction distinguishes a class of differential operators for which we study a density of states.
Joint with: Nathan Hannon and Jerome Kaminker.

Anders Björner (KTH Stockholm) and **Karim Adiprasito** (IHES, Paris)

Title: Filtered geometric lattices and tropical Lefschetz theorem.

In the first part of this talk we prove a conjecture of Mikhalkin and Ziegler concerning positive sum systems in geometric lattices. This builds on methods from topological combinatorics.

In the second part of the talk we establish analogues for smooth tropical varieties of the classical Lefschetz Section Theorem. For this the result from part one on geometric lattices provides a crucial index estimate for the stratified Morse data at critical points of the tropical variety, which we consider as a Whitney stratified space.

Thomas Brady (Dublin City University)

Title: Non-crossing partitions and monodromy of Milnor Fibres.

Abstract: For arrangements associated to finite real reflection groups, we determine the action of the monodromy on simplicial models for the Milnor fibres. This is joint work with Mike Falk and Colum Watt.

Mike Davis (Ohio State University)

Title: Complements of hyperplane arrangements as posets of spaces.

Abstract: The complement of an affine hyperplane arrangement is a poset of spaces, where the indexing poset is the intersection poset and where the space corresponding to an intersection is the complement of the corresponding central arrangement. An application of this gives a spectral sequence for computing cohomology.

Emanuele Delucchi (University of Fribourg)

Title: Recent developments in toric arrangements.

Abstract: The study of toric arrangements is rooted in the literature in both its topological (since at least Looienga in 1993) and combinatorial aspects (e.g., Ehrenborg, Readdy and Slone 2009). Recent work of De Concini, Procesi and Vergne provided a fresh impulse towards a comprehensive study of this subject, viewed as a generalization of the successful theory of hyperplane (or subspace) arrangements in vector spaces. Out of this impulse grew many new results and techniques, concerning both topology and in combinatorics, which I will try to survey with an eye towards setting up a general combinatorial-topological framework which might lead to the treatment of even more general types of arrangements. Some of the results I will present have been obtained in joint works with Karim Adiprasito, Filippo Callegaro, Giacomo d'Antonio or Sonja Riedel.

Graham Denham (University of Western Ontario)

Title: Elliptic braid groups are duality groups.

Abstract: The elliptic braid group is the fundamental group of a configuration space of n points in 2-dimensional torus. We show that such groups are duality groups, extending the known result for classical braid groups. The method is an instance of a more general cohomological vanishing construction which also has applications to torus arrangements, right-angled Artin groups, and hyperplane complements. This is joint work with Alex Suciu (Northeastern) and Sergey Yuzvinsky (Oregon).

Michael J. Falk (Northern Arizona University)

Title: BGG resolutions and configuration spaces.

Abstract: Let $\mathcal{A}$ be a projective arrangement and $p\colon X \to \mathbb{P}^k$ the minimal resolution of $\mathcal{A}$. Let D be a union of components of the normal-crossing divisor $p^{-1}(\bigcup \mathcal{A})$, and let $\mathcal{L}$ be a rank-one local system on $X - D$. We present a complex $A^{\cdot}(X, D; \mathcal{L})$, built out of Aomoto complexes of minors of $\mathcal{A}$ and residue maps between them, that computes the cohomology $H^*(X - D, \mathcal{L})$. When $\mathcal{A} = \mathcal{A}_{k,n}$ is an sl$_2$-discriminantal arrangement, and $\mathcal{L}$ is determined by a collection $L_{m_1}, \ldots, L_{m_n}$ of irreducible finite-dimensional sl$_2$ modules, the skew-symmetric part of the complex $A^{\cdot}(X, D; \mathcal{L})$ is isomorphic to the Bernstein-Gelfand-Gelfand resolution of $L_{m_1} \otimes \cdots \otimes L_{m_n}$. We illustrate with some examples.
This is joint work with V. V. Schechtman and A. N. Varchenko.

Michael Farber (University of Warwick)

Title: Large random spaces and groups.

Abstract: I will discuss several models producing random simplicial complexes. We are interested in geometric and topological properties of random simplicial complexes which are satisfied with probability tending to 1 as the number of vertices of the complex tends to infinity. In the talk I will explain why for random simplicial complexes the Whitehead and Eilenberg-Ganea Conjectures hold. I shall also describe torsion and the cohomological dimension of fundamental groups of random simplicial complexes. This is a joint work with Armindo Costa.

Matthias Franz (University of Western Ontario)

Title: Big polygon spaces.

Abstract: Polygon spaces are configuration spaces of polygons with prescribed edge lengths. We present a related family of spaces, called big polygon spaces. They come with canonical torus actions, whose fixed point sets are polygon spaces. It turns out that big polygon spaces provide examples of torus actions on compact orientable manifolds whose existence has previously been an open question. For example, while the equivariant cohomology of a big polygon space is never free, it can be computed via the "GKM method", and the equivariant Poincaré pairing is perfect. More generally, big polygon spaces show that a certain bound obtained by Allday-Franz-Puppe and concerning syzygies in torus-equivariant cohomology is optimal.

Dmitry Kozlov (University of Bremen)

Title: Configuration spaces arising in distributed computing.

Abstract: In this talk we shall describe a family of simplicial complexes, called protocol complexes, which arise naturally as some of the central objects in the field of theoretical distributed computing. These complexes give a description of the totality of all possible executions of distributed protocols in a fixed computational model. They are the natural analog of configuration spaces in this context.

Part of the talk will be based in the recent book "Distributed Computing through Combinatorial Topology", joint with M. Herlihy and S. Rajsbaum.

Gus Lehrer (University of Sydney)

Title: Cohomology of the Milnor fibre of an arrangement.

Abstract: I will discuss problems concerning the cohomology of the Milnor fibre of a hyperplane arrangement, both generally and in the case of the set of reflecting hyperplanes of a unitary reflection group. The basic problem is to determine the Hodge-Deligne polynomials, equivariantly, but this is currently a distant dream. I shall report on some results with Dimca, relating cohomology degree and Hodge structure to the order of the monodromy, as well as some more particular results.

Jon McCammond (University of California, Santa Barbara)

Title: The structure of euclidean Artin groups.

Abstract: The Coxeter groups that act geometrically on euclidean space have long been classified and presentations for the irreducible ones are encoded in the well-known extended Dynkin diagrams. The corresponding Artin groups are called euclidean Artin groups and, despite what one might naively expect, most of them have remained fundamentally mysterious for more than forty years. Recently, my coauthors and I have resolved several long-standing conjectures about these groups, proving for the first time that every irreducible euclidean Artin group is a torsion-free centerless group with a decidable word problem and a finite-dimensional classifying space. In my talk I will survey our results and the techniques we use to prove them.

Luca Moci (University of Paris 7)

Title: Matroids over a ring: motivations, examples, perspectives.

Abstract: Several objects can be associated to a list of vectors with integer coordinates: a toric arrangement, a zonotope, a vector partition function. The linear algebra of the list is encoded by the notion of a matroid, but several properties of the objects above depend also on the arithmetics of the list: this is retained by the notion of a "matroid over $\mathbb{Z}$". Similarly, applications to tropical geometry suggest to consider matroids over a discrete valuation ring. Motivated by these examples, we introduce the notion of a "matroid over a commutative ring R". When R is a Dedekind domain, we can extend the usual properties and operations holding for matroids, and compute the Tutte-Grothendieck ring of matroids over R; the class of a matroid in such a ring specializes to several known invariants. We will outline possible applications and open problems. (Based on joint work with A. Fink)

Claudio Procesi (Università di Roma I)

Title: From the theorem of Amitsur-Levitzki to exterior algebras of simple Lie algebras.

Abstract: The classical theorem of Amitsur-Levitzki tells us that the algebra on $n \times n$ matrices satisfies the standard polynomial in $2n$ variables, the essentially unique alternating multilinear non-commutative polynomial in degree $2n$.
We revisit this theorem in several ways, analyzing the structure of multilinear alternating and equivariant maps, first for matrices of various kinds and then in general for simple Lie algebras. The results obtained are quite precise and give the general framework for Theorems of type similar to Amitsur-Levitzki.

Hal Schenck (University of Illinois)

Title: Chen ranks and resonance.

Abstract: The Chen groups of a group G are the lower central series quotients of the maximal metabelian quotient of G. Under certain conditions, we relate the ranks of the Chen groups to the first resonance variety of G, a jump locus for the cohomology of G. In the case where G is the fundamental group of the complement of a complex hyperplane arrangement, our results positively resolve Suciu's Chen ranks conjecture. We obtain explicit formulas for the Chen ranks of a number of groups of broad interest, including pure Artin groups associated to Coxeter groups, and the group of basis-conjugating automorphisms of a finitely generated free group. (joint work with Dan Cohen, Lousiana State University)

Alex Suciu (Northeastern University)

Title: Topology of line arrangements.

Abstract: I will discuss some recent advances in our understanding of the relationship between the topology, group theory, and combinatorics of a complex line arrangement.

Michele Vergne (University of Paris 7)

Title: Euler Mac Laurin formula for rational polytopes.

Abstract: Let P be a polytope defined by rational inequalities. We consider a smooth function and we give an aysmptotic formula for summing the function on points in P belonging to a rescaled lattice. This formula generalize the local Euler-Mac Laurin formula defined by Baldoni-Berline-Vergne for polynomial functions. (Common work with Nicole Berline)

Sergey Yuzvinsky (University of Oregon)

Title: Higher topological complexity of Eilenberg-MacLane spaces.

Abstract: Topological complexity $TC(X)$ of a topological space X was introduced by M. Farber about 10 years ago as a specialization of the Schwarz genus depending only on the homotopy type of X. About 5 years ago, Yu. Rudyak generalized Farber's definition to obtain higher (s-th) topological complexity $TC_s(X)$ for $s \geq 2$ that coincides with $TC(X)$ for $s = 2$. For X being the complement of a complex arrangement of hyperplanes there were previous attempts to calculate $TC(X)$. They were successful for particular classes of arrangements such as Coxeter infinite series (Farber and Y) and generic arrangements (Y, Cohen and Pruidze). In a recent paper by Gonzalez and Grant, TC_s was computed (for an arbitrary s) for the Coxeter series of type A. In the talk, we will give a simple combinatorial condition that allows us to calculate TC_s for all complex reflection arrangements, i.e., for a class of $K(\pi, 1)$ spaces. If time allows we will talk about new upper and lower bounds for TC_s for arrangement complements and TC_s for generic arrangements.

TITLES AND ABSTRACTS OF POSTERS

Samia Ashraf (Government College University of Lahore)

Title: About cohomology of configuration spaces of smooth complex projective varieties.

Abstract: For X a smooth complex projective variety, the Kriz model $E(X, n)$ is a rational model for the ordered n-points configuration space $F(X, n)$. The natural action of the symmetric group S_n on $F(X, n)$ induces an action on the model $E(X, n)$. We study this S_n action on $E(X, n)$ and describe an S_n equivariant acyclic subcomplex of the model. We also compute the Betti numbers and describe the cohomology algebras of the ordered and unordered configuration spaces of three points in complex projective spaces $\mathbb{CP}^m$.

Haniya Azam (Government College University of Lahore)

Title: Kriz model and the cohomology of configuration spaces.

Abstract: The Kriz model, which is a rational model for configuration spaces is a useful tool for computing their cohomology. We apply some S_n representation theory to obtain cohomology for configuration spaces of fewer points on Riemann surfaces.

Jeremiah Bartz (Francis Marion University)

Title: Induced Multinets in P^2

Abstract: Multinets are specific configurations of points and lines in P^2 which play an important role in the study of resonance varieties of complex hyperplane arrangement complements. Very few examples were known until recently. The new examples were obtained from nets in P^3. In this poster, examples and known properties about these induced multinets are presented.

Christin Bibby (University of Oregon)

Title: Cohomology of abelian arrangements.

Abstract: An abelian arrangement is a finite set of codimension one abelian subvarieties (possibly translated) in a complex abelian variety. Here, we study the cohomology of the complement of an abelian arrangement, denoted by $M(\mathcal{A})$. For unimodular abelian arrangements, we provide a combinatorial presentation for a differential graded algebra whose cohomology is isomorphic to the rational cohomology of $M(\mathcal{A})$. This DGA is also a model for the space $M(\mathcal{A})$ in the sense of rational homotopy theory. Moreover, this DGA has a bi-grading that allows us to compute the mixed Hodge numbers.

Elizabeth Leyton Chisholm (University of California, Santa Barbara)

Title: Braid groups and euclidean simplices.

Abstract: The Lawrence-Krammer-Bigelow representation is a faithful linear representation of the braid group in two variables q and t that are closely related to two different Garside structures on the braid group. The t variable is related to the standard Garside structure and it is this structure that was used to prove linearity. The q variable is related to the dual Garside structure and the role it plays in the LKB representation has received less attention.

In this poster we give a geometric interpretation of the q part of the LKB representation in terms of an action of the braid group on the space of euclidean simplices. In particular every dual simple element in the dual Garside structure of the braid group has an elegant interpretation as a way to systematically reshape euclidean simplices.

Sonja Riedel (University of Bremen)

Title: What is a toric pseudoarrangement?

Abstract: A toric arrangement is a finite family $\mathcal{A}$ of special subtori of codimension one in the torus $(C^*)^n$ or $(S^1)^n$. Recent work of De Concini and Procesi generated new interest in combinatorial invariants of the topology of the complement of a toric arrangement. In the case of so-called 'complexified' toric arrangements, the induced decompostion of the compact torus $(S^1)^n$ into faces determines the homotopy type of the complement (via the toric Salvetti complex). Starting from the theory of semimatroids and oriented matroids, we want to develop a toric oriented matroid in order to characterize appropriate face structures on the compact torus. With a view towards generalizing the topological results about the complexified complement, using the toric Salvetti complex associated to the face categories of such *toric pseudoarrangements*.

Elia Saini (University of Bremen)

Title: Phasing classes of matroids.

Abstract: Phased matroids were introduced by Anderson and Delucchi as a generalization of oriented matroids to the complex setting. In order to understand the set $\mathcal{M}_{\mathbb{C}}(M)$ of phased matroids with given underlying matroid M we define and study the set $\mathcal{R}_{\mathbb{C}}(M)$ of phasing classes of phased matroids over M. We generalize methods of Gel'fand, Rybnikov and Stone and we obtain three different characterizations of $\mathcal{R}_{\mathbb{C}}(M)$:

1. A combinatorial one, via phased projective orientations defined in terms of the ciruits and cocircuits of M;

2. An algebraic one as a subset of group homomorphisms from the inner Tutte group $\mathbb{T}_{M(0)}$ to S^1;

3. A geometric one as a subspace of the torus $(S^1)^d$, which allows us to explicitly compute some examples.