Numerical methods for tridiagonal quadratic eigenvalue problems

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We consider a tridiagonal hyperbolic quadratic eigenvalue problem (QEP)

$$Q(\lambda)x = (\lambda^2 M + \lambda C + K)x = 0,$$

where $M, C$, and $K$ are $n \times n$ tridiagonal real symmetric matrices, $M$ is positive definite and

$$(x^*Cx)^2 > 4(x^*Mx)(x^*Kx)$$

for all $x \neq 0$. Our goal is to compute all or some of the eigenvalues, i.e. scalars $\lambda$ such that $\det(Q(\lambda)) = 0$. A hyperbolic QEP has $2n$ real eigenvalues and eigenvectors and all eigenvalues are semisimple. There is a gap between $n$ largest and $n$ smallest eigenvalues. We show that if the QEP $Q$ is hyperbolic then the inertia of a symmetric matrix $Q(\sigma)$ is related to the number of the eigenvalues of the QEP $Q$ that are larger or smaller than $\sigma$.

Eigenvalues are computed as zeros of the characteristic polynomial using bisection, Laguerre's method, the Ehrlich-Aberth method, and the Durand-Kerner method. Good initial approximations are provided by the recursive use of divide and conquer approach using rank two modifications.

The above methods require stable and efficient computation of $f(\lambda)$, $f'(\lambda)/f(\lambda)$ and $f''(\lambda)/f(\lambda)$, where $f(\lambda) = \det(Q(\lambda))$. We discuss how to obtain these values using the three term recurrences, QR decomposition, and LU decomposition.

We show that some of the presented methods can be applied to more general problems, for example, to the non hyperbolic tridiagonal QEPs, banded polynomial eigenvalue problems, etc.