

Polynomials

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Exercise 0.1. *Compute the following divisions*

1. $(x^4 - 5x^3 + x - 1)/(x - 3)$

Sol: Record[Quotients = $[x^3 - 2x^2 - 6x - 17]$, Remainder = -52]

2. $(x^5 - 5x^3 + 2x - 1)/(x^3 - 3x)$

Sol: Record[Quotients = $[x^2 - 2]$, Remainder = $-4x - 1$]

3. $(x^5 - 5x^3 + 2x - 1)/(x^2 - x - 1)$

Sol: Record[Quotients = $[x^3 + x^2 - 3x - 2]$, Remainder = $-3x - 3$]

4. $(x^3 - 2x - 1)/(2x - 1)$

Sol: Record[Quotients = $[1/2x^2 + 1/4x - 7/8]$, Remainder = $-15/8$]

5. $(x^3 - 2x^2 - 1)/(x - 1/2)$

Sol: Record[Quotients = $[x^2 - 3/2x - 3/4]$, Remainder = $-11/8$]

Exercise 0.2. *Solve the gcd*

1. $\gcd((x - 2)^3(x + 1)^2, ((x - 2)^2)(x^2 + x + 1)^2)$ *Sol:* $(x - 2)^2$

2. $\gcd(x^3 - 2, x^2 + 1)$ *Sol:* 1

3. $\gcd(x^3 - 3x^2 + x - 3, x^3 - 2x^2 + x - 2)$ *Sol:* $x^2 + 1$

4. $\gcd(x^3 + 4x^2 + x - 6, x^3 + 6x^2 + 11x + 6)$ *Sol:* $x^2 + 5x + 6$

Exercise 0.3. *Solve the equations*

1. $x^4 - 3x^3 - 9x^2 - 3x - 10 = 0$. *Sol:* $x = -2, 5$

2. $x^4 - 3x^2 - 10 = 0$. *Sol:* $x = \pm\sqrt{5}$

3. $x^4 - 3x^2 - 4 = 0$. *Sol:* $x = \pm 2$

4. $x^3 + 12x^2 + 29x - 42 = 0$. *Sol:* $x = 1, -6, -7$

Exercise 0.4. *Solve the systems*

$$1. \begin{cases} x^3 - 6x^2 + 11x - 6 = 0 \\ x^3 - x^2 + x - 1 \end{cases} \quad . \text{ Sol: } x = 1$$

$$2. \begin{cases} x^3 - 6 = 0 \\ x^5 - 1 \end{cases} \quad . \text{ Sol: } \emptyset$$