Math 0 Final - 2025

Dear online students, please send the solutions to me by email to the address `massimo.caboara@unipi.it` no later than 18:00. No HEIC files are allowed.

In your answers please show all the necessary reasoning and steps AND MARK CLEARLY THE SOLUTIONS THEMSELVES. You will receive your Final and total grades by Thursday, October 30th.

The students taking the Short final have to solve only the (a) questions, the other have to solve all the questions.

1. (a) Solve the inequality $\left|x^2 - 1\right| > 3$

   Solution. $x \in (-\infty, -2) \cup (2, +\infty)$ □

   (b) Solve the equation $|x + 2| - |x - 1| = 2$

   Solution. $x = \frac{1}{2}$ □

2. (a) Draw the graph of the function $\begin{array}{rccc} F\colon & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & |x+2|+1 \end{array}$

   Solution. The graph of the absolute value, shifted left by 2 and up by 1 □

   (b) Draw the graph of the function $\begin{array}{rccc} F\colon & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & (x+1)^2-2 \end{array}$

   Solution. The standard parabola, shifted left by 1 and down by 2 □

3. (a) Shrink the domain/codomain of the function $\begin{array}{rccc} F\colon & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & |x+2|+1 \end{array}$ to make it one-to-one.

   Solution. Domani: $(-2, +\infty)$. Codomain: $(1, +\infty)$ □

   (b) Shrink the domain/codomain of the function $\begin{array}{rccc} F\colon & \mathbb{R} & \to & \mathbb{R} \\ & x & \mapsto & (x+1)^2-2 \end{array}$ to make it surjective but not one-to-one.

   Solution. Domani: $\mathbb{R}$. Codomain: $(-2, +\infty)$ □

4. (a) Solve the equation $\log_3(x + 3) + \log_3(x) = \log_3(x^2 + x + 3)$

   Solution. $x = \frac{3}{2}$ □

   (b) Solve the inequality $\log_3(|x|) < 0$

   Solution. $x \in (-1, 0) \cup (0, 1)$ □

5. (a) Compute $\gcd(3579, 129)$

   Solution. 3 □

   (b) Compute $\gcd(2247, 1512)$

   Solution. 21 □

6. (a) Solve the equation $10^{2x+1} = 3^{2-3x}$

   Solution. $x = -\frac{\log_{10} \frac{10}{9}}{\log_{10} 2700}$ □

   (b) Solve the inequality $49^{3x-1} + 19^{2-3x} > 0$

   Solution. All $x \in \mathbb{R}$ □

7. (a) Solve the equation $x^4 - x^2 - 2 = 0$

   Solution. $x = \pm\sqrt{2}$ □

   (b) Solve the inequality $x^4 - x^2 - 2 > 0$

   Solution. $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$ □

8. (a) Solve the inequality $\frac{x^2+3x+2}{x^2-1} > 0$

   Solution. $x \in (-\infty, -2) \cup (1, +\infty)$ □

   (b) Solve the inequality $\frac{x^2+5x+6}{x^2-5x+6} < 0$

   Solution. $x \in (-3, -2) \cup (2, 3)$ □