

Classwork: Points, Lines, Parabolas and Circles

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Time: 30 minutes

Name: _____ Student ID: _____

Useful formulas.

- Distance: $d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Circle with center $C(x_0, y_0)$ and radius r : $(x - x_0)^2 + (y - y_0)^2 = r^2$.
- For $x^2 + y^2 + ax + by + c = 0$: center $C(-\frac{a}{2}, -\frac{b}{2})$, $r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4} - c}$ (a circle only if the quantity under the root is positive).

Exercise 1. Compute the distance between $A(1, -3)$ and $B(7, 5)$.

Solution. $x_2 - x_1 = 7 - 1 = 6$, $y_2 - y_1 = 5 - (-3) = 8$.

$$d(A, B) = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$$

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Exercise 2. Write the equation of the circle with center $C(2, -3)$ and radius $r = 4$.

Solution. Step 1: standard form. $(x - 2)^2 + (y + 3)^2 = 16$.

Step 2: expand. $x^2 - 4x + 4 + y^2 + 6y + 9 = 16$.

Step 3: general form. $x^2 + y^2 - 4x + 6y - 3 = 0$.

Check: $a = -4$, $b = 6$, $c = -3$ give $C(2, -3)$ and $r = \sqrt{4 + 9 + 3} = 4$.

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Exercise 3. Find center and radius of the circle $x^2 + y^2 + 10x - 4y + 20 = 0$.

Solution. Step 1: complete the squares. $x^2 + 10x = (x + 5)^2 - 25$, $y^2 - 4y = (y - 2)^2 - 4$, so

$$(x + 5)^2 - 25 + (y - 2)^2 - 4 + 20 = 0 \implies (x + 5)^2 + (y - 2)^2 = 9.$$

Step 2: read off. Since $9 > 0$: center $C(-5, 2)$, radius $r = 3$.

With the formulas: $a = 10$, $b = -4$, $c = 20$: $C(-5, 2)$, $r^2 = 25 + 4 - 20 = 9$.

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Exercise 4. Find the intersection points of the circle $x^2 + y^2 = 25$ and the line $y = x + 1$.

Solution. Step 1: substitute $y = x + 1$ into the circle.

$$x^2 + (x + 1)^2 = 25 \implies x^2 + x^2 + 2x + 1 = 25 \implies 2x^2 + 2x - 24 = 0.$$

Step 2: solve. Dividing by 2: $x^2 + x - 12 = 0$, with $\Delta = 1 + 48 = 49 > 0$:

$$x = \frac{-1 \pm 7}{2} \implies x_1 = 3, x_2 = -4.$$

Step 3: find y . $y_1 = 3 + 1 = 4$, $y_2 = -4 + 1 = -3$.

Conclusion. Two intersection points: $(3, 4)$ and $(-4, -3)$.

Check: $9 + 16 = 25$ and $16 + 9 = 25$.

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Exercise 5. Find the intersection points of the circle $x^2 + y^2 = 13$ and the parabola $y = x^2 - 7$.

Solution. Step 1: express x^2 from the parabola. $x^2 = y + 7$, with the condition $y \geq -7$.

Step 2: substitute into the circle.

$$(y + 7) + y^2 = 13 \implies y^2 + y - 6 = 0.$$

Step 3: solve. $\Delta = 1 + 24 = 25$, $y = \frac{-1 \pm 5}{2}$, so $y_1 = 2$, $y_2 = -3$.

Step 4: check the condition $y \geq -7$ and find x .

- $y_1 = 2$: $x^2 = 9 \Rightarrow x = \pm 3$: the points $(3, 2)$, $(-3, 2)$;
- $y_2 = -3$: $x^2 = 4 \Rightarrow x = \pm 2$: the points $(2, -3)$, $(-2, -3)$.

Conclusion. Four intersection points: $(3, 2)$, $(-3, 2)$, $(2, -3)$, $(-2, -3)$.

Check: $9 + 4 = 13$ and $4 + 9 = 13$.

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