

$K[x]$
 ≤ 2

$\text{Span}(1, x, x^2)$

PAR.

$\text{Span}(x^2+x, x+1, 5)$

$\alpha, \beta, \gamma \in K$
W.C.

$a, b, c \in K$

$$\alpha(x^2+x) + \beta(x+1) + \gamma 5 \equiv \alpha x^2 + b x + c$$

$$\alpha x^2 + (\alpha + \beta)x + \beta + 5\gamma \equiv \alpha x^2 + b x + c$$

\Leftrightarrow

$$\alpha = a$$

$$\alpha + \beta = b$$

$$\beta + 5\gamma = c$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 1 & 1 & 0 & b \\ 0 & 1 & 5 & c \end{array} \right]$$

$$\text{rk}(W.C.) = 3 \Rightarrow \text{rk}(CUMPL) = 3$$

\Downarrow
 \Rightarrow ! suc R-C

$$A' \quad \alpha = \kappa = 0$$

$$\left[\begin{array}{cc|c} 0 & 1 & 2 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & 2 & -1 \end{array} \right]$$

A

$$\text{rk}(A) = 2$$

$$\text{rk}(A') = 2$$



$\exists \infty'$ sol

$$\begin{bmatrix} 2(k-1) & k-1 & k-3 & -6 \\ 2(k-1) & 3k+2 & 4k & -11 \\ k-1 & -1 & -2 & -2 \end{bmatrix} \quad \begin{array}{l} \text{SE } k=1 \\ \text{SOSTITUISCO} \end{array}$$

↓

$k \neq 1$

$$\begin{bmatrix} 2 & k-1 & k-3 & 6 \\ 2 & 3k+2 & 4k & 11 \\ 1 & -1 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -2 & 2 \\ 0 & k+1 & k+1 & 2 \\ 0 & 3k+4 & 4k+4 & 7 \end{bmatrix}$$

A

$$\det(A) = 4(k+1)(k+1) - (k+1)(3k+4)$$

$$(k+1)(4k+4 - 3k - 4) \quad k=0 \text{ SOST}$$

$$(k+1)(k) \quad k=-1 \text{ SOST}$$

$$k \neq 0, -1 \quad \det(A) = 3$$

$$\downarrow$$

$$\text{rn}(A) = 3$$

↓ R_c

∴! SOL

$$\mathbb{Q} \subsetneq \mathbb{Q}[\sqrt{2}] \subsetneq \mathbb{R} \subsetneq \mathbb{C}$$

=

$$\{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$$

$$b=0 \quad \text{H.O.} \quad \mathbb{Q} \quad \subseteq$$

$$\sqrt{2} \notin \mathbb{Q} \quad \subsetneq$$