(V) Prodotto misto – numero

esiste solo nello spazio!!!

① $(\vec{a}, \vec{b}, \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$

② $\vec{a}(a_1, a_2, a_3)$; $\vec{b}(b_1, b_2, b_3)$; $\vec{c}(c_1, c_2, c_3)$

$$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\vec{a}(2,-1,3)$ $\quad \vec{b}(0,-1,4)$ $\quad \vec{c}(1,1,1)$

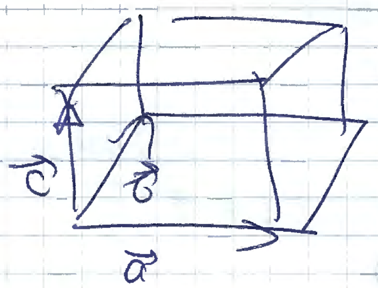
$$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} 2 & -1 & 3 \\ 0 & -1 & 4 \\ 1 & 1 & 1 \end{vmatrix} = -2+0-4-(-3+8+0) =$$
$$= -6-5 = -11$$

③ Se $[\vec{a}, \vec{b}, \vec{c}] \subset \alpha$ (sono complanari)

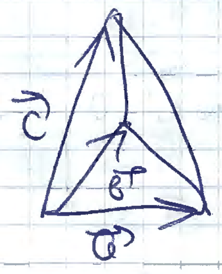
$\Rightarrow (\vec{a}, \vec{b}, \vec{c}) = 0$

Se $(\vec{a}, \vec{b}, \vec{c}) = 0 \Rightarrow$ $\vec{a}, \vec{b}, \vec{c}$ sono complanari $(\vec{a}, \vec{b}, \vec{c} \neq \vec{0})$
o
almeno uno dei vettori è nullo

④



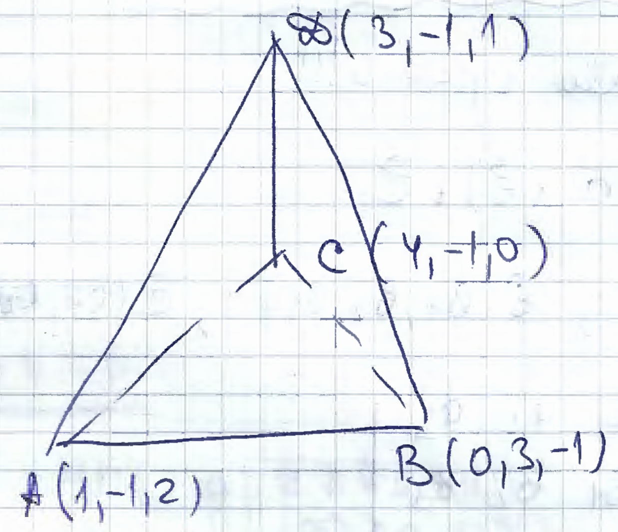
$V_{paralelopipedo} = |(\vec{a}, \vec{b}, \vec{c})|$



$V = \frac{1}{6} |(\vec{a}, \vec{b}, \vec{c})|$

↓ piramide triangolare

1 Trovare il volume del piramide triangolare ABCD =22=



$\vec{AB}(-1,4,-3)$ $\qquad (\vec{AB},\vec{AC},\vec{AD}) = \begin{vmatrix} -1 & 4 & -3 \\ 3 & 0 & -2 \\ 2 & 0 & -1 \end{vmatrix} =$

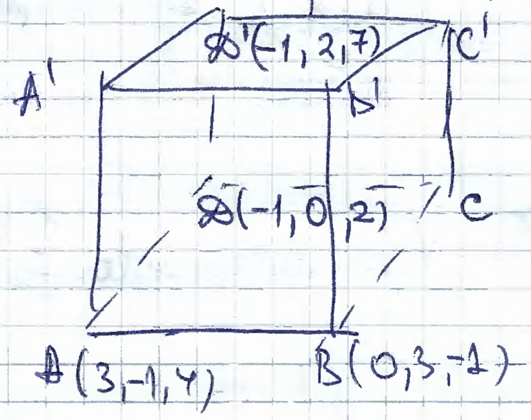
$\vec{AC}(3,0,-2)$

$\vec{AD}(2,0,-1)$

$= -16+12 = -4$

$V_{ABCD} = \frac{1}{6} \cdot |-4| = \frac{4}{6} = \frac{2}{3}$

2 Trovare il volume del parallelepipedo ABCDA'B'C'D'



$V_{ABCDA'B'C'D'} = |(\vec{AB},\vec{AD},\vec{AA'})|$

Sia $A'(x,y,z)$ $\qquad \vec{AA'}(x-3,y+1,z-4) = \vec{DD'}(0,2,5)$

$\Rightarrow x-3=0 \quad x=3$

$y+1=2 \quad y=1 \quad \rightarrow A'(3,1,9)$

$z-4=5 \quad z=9$

non è necessario, perché $\vec{AA'} = \vec{DD'}$)

$\Rightarrow V = |(\vec{AB}, \vec{AD}, \vec{DD'})|$

$\vec{AB}(-3, 4, -5)$

$\vec{AD}(-4, 1, -2)$

$\vec{DD'}(0, 2, 5)$

$(\vec{AB}, \vec{AD}, \vec{DD'}) = -15 + 40 - 12 + 80 = 120 - 27 = 93$

$V = |103| = 93$

Equazioni delle curve nel piano

I La retta

1 $g \begin{cases} \ni M_0(x_0, y_0) \\ \parallel \vec{p}(a, b) \neq \vec{0} \end{cases}$ $\quad g: \dfrac{x - x_0}{a} = \dfrac{y - y_0}{b}$

1) $g \begin{cases} \ni M_0(2, -3) \\ \parallel \vec{p}(-7, 4) \end{cases}$ $\quad g: \dfrac{x-2}{-7} = \dfrac{y+3}{4}$

$g: 4x - 8 = -7y - 21$

$\underline{g: 4x + 7y + 13 = 0}$

2) $g \begin{cases} \ni M_0(-1, 4) \\ \parallel \vec{p}(3, 0) \end{cases}$ $\quad g: \dfrac{x+1}{3} = \dfrac{y-4}{0}$

non è divisione significa che $\boxed{y - 4 = 0}$

$\underline{g: y - 4 = 0}$

2 $g \begin{cases} \ni A \\ \ni B \end{cases} \Rightarrow g \begin{cases} \ni A \\ \parallel \vec{AB} \end{cases}$ caso 1

=24=

1) $g\left|\begin{array}{l}\ni A(-1,2)\\ \ni B(3,4)\end{array}\right. \Rightarrow g\left|\begin{array}{l}\ni A(-1,2)\\ \parallel \overrightarrow{AB}(4,2)\end{array}\right.$

$g: \dfrac{x+1}{4} = \dfrac{y-2}{2}$

$g: 2x+2 = 4y-8$

$g: 2x-4y+10=0 \quad |:2$

$\underline{g: x-2y+5=0}$

2) $g\left|\begin{array}{l}\ni A(3,-11)\\ \ni B(3,2)\end{array}\right. \Rightarrow g\left|\begin{array}{l}\ni A(3,-11)\\ \parallel \overrightarrow{AB}(0,13)\end{array}\right.$

$g: \dfrac{x-3}{0} = \dfrac{y+11}{13}$

$\underline{g: x-3=0}$

3) $g\left|\begin{array}{l}\ni M_0(x_0,y_0)\\ \perp \overrightarrow{N}(A,B)\end{array}\right. \qquad g: A(x-x_0)+B(y-y_0)=0$

1) $g\left|\begin{array}{l}\ni A(-3,1)\\ \perp \overrightarrow{N}(2,-7)\end{array}\right. \qquad g: 2(x+3)-7(y-1)=0$

$\underline{g: 2x-7y+13=0}$

4) $g\left|\begin{array}{l}\ni M_0(x_0,y_0)\\ \parallel \overrightarrow{p}(a,b)\neq\overrightarrow{0}\end{array}\right.$

Equazioni parametriche $\quad \left|\begin{array}{l}x = x_0+at\\ y = y_0+bt\end{array}\right.$

$t\in\mathbb{R}$ o $t\in(-\infty,+\infty)$

1) $g \left| \begin{array}{l} \ni A(-1,3) \\ \parallel \vec{p}(7,-2) \end{array} \right.$ $\quad g: \left| \begin{array}{l} x = -1 + 7t \\ y = 3 - 2t \end{array} \right.$ $t \in R$

2) $g \left| \begin{array}{l} \ni A(2,-1) \\ \ni B(-1,-1) \end{array} \right.$ $\quad g \left| \begin{array}{l} \ni A(2,-1) \\ \parallel \vec{AB}(-3,0) \end{array} \right.$ $\quad g: \left| \begin{array}{l} x = 2 - 3t \\ y = -1 \end{array} \right.$ $t \in R$

3) $g \left| \begin{array}{l} \ni A(0,-1) \\ \ni B(0,2) \end{array} \right.$ $\quad g \left| \begin{array}{l} \ni A(0,-1) \\ \parallel \vec{AB}(0,3) \end{array} \right.$ $\quad g: \left| \begin{array}{l} x = 0 \\ y = -1 + 3t \end{array} \right.$ $t \in R$

4) $g: 2x - 3y + 1 = 0$ $\quad \left| \begin{array}{l} x = t \\ y = \frac{2}{3}t + \frac{1}{3} \\ t \in R \end{array} \right.$ $\Rightarrow 2t - 3y + 1 = 0 \quad 3y = 2t + 1$

5) $g: x = 0$ $\quad \left| \begin{array}{l} x = 0 \\ y = t \end{array} \right.$ $t \in R$

6) $g: y = 0$ $\quad \left| \begin{array}{l} x = t \\ y = 0 \end{array} \right.$ $t \in R$

5 $\quad g: \; Ax + By + C = 0$ dove $|A| + |B| \neq 0$

$\Rightarrow \vec{p}(-B, A) \parallel g$

$\vec{N}(A, B) \perp g$

$\boxed{1}$ Scrivete le equazioni parametriche della retta g, dove

a) $g \left| \begin{array}{l} \ni M(-1,2) \\ \perp p: 2x - 3y + 1 = 0 \end{array} \right.$

$\Downarrow$

$\vec{N}(2,-3) \perp p \Rightarrow \parallel g$

b) $g \left| \begin{array}{l} \ni A(1,-1) \\ \parallel p: 3x + y - 1 = 0 \end{array} \right.$

$\parallel \vec{p}(-1,3) \parallel p \Rightarrow$

$\vec{p} \parallel g$

c) $g \begin{cases} \ni p.S \text{ dove } S = p \cap q \\ \perp q \end{cases}$ $p: x-y+2=0$
$q: 3x-1+y=0$

~~Svolg~~. 1) $\begin{cases} x-y+2=0 \\ 3x+y-1=0 \end{cases}$ $\Rightarrow y = x+2$
$3x+x+2-1=0 \quad 4x=-1 \quad x=-\frac{1}{4}$
$y = 2-\frac{1}{4}=\frac{7}{4}$

2) $\Rightarrow p.S\left(-\frac{1}{4}, \frac{7}{4}\right)$

3) $\vec{N}(3,1) \perp q \Rightarrow g \parallel \vec{N}$

2 Scrivere una parametrizzazione dei segmenti ~~seguenti~~ aventi per ~~estremi~~ estremi le seguenti coppie di punti

a) $A(-1,2)$ $B(0,-3)$

$AB \begin{cases} \ni A(-1,2) \\ \parallel \vec{AB}(1,-5) \end{cases}$ $AB: \begin{cases} x=-1+t \\ y=2-5t \end{cases}$ $t \in [0,1]$

$x=-1+t \quad x_A=-1 \quad -1+t=-1 \quad t=0$
$x_B=0 \quad -1+t=0 \quad t=1$

b) Punto di partenza $A(3,-1)$
Punto di arrivo $B(-1,4)$

$AB \begin{cases} \ni A(3,-1) \\ \parallel \vec{AB}(-4,5) \end{cases}$ $AB \begin{cases} x = 3-4t \\ y=-1+5t \end{cases}$

$3-4t=3 \quad t=0$
$3-4t=-1 \quad t=1$ $\Rightarrow t \in [0,1]$ o $0 \le t \le 1$

t è dallo 0 alle 1

c) Punto di partenza $B(-1,-1)$
Punto di arrivo $A(1,0)$ $AB \begin{cases} \ni A(1,0) \\ \parallel \vec{AB}(-2,-1) \end{cases}$

$AB: \begin{cases} x=1-2t \\ y=-t \end{cases}$ $y=-t$ $y_B=-1 \quad t=1$
$y_A=1 \quad t=-1$

$1 \ge t \ge -1$ dalle 1 alle -1

c) $A(-1,1)$ $B(0,2)$

d) $A(-3,1)$ $O(0,0)$

e) $A(2,-1)$ $B(3,-1)$ Partenza B Arrivo A

f) $L(-1,0)$ $S(0,1)$ Partenza L Arrivo S

II La parabola

γ_1: $(ax+b)^2 = cy+d$ dove $a>0$; $c\neq0$

$P\left(-\frac{b}{a}, -\frac{d}{c}\right)$ vertice

l'asse Oy $c>0$ ∪

$c<0$ ∩

γ_2: $(ay+b)^2 = cx+d$ dove $a\geq0$; $c\neq0$

$P\left(-\frac{d}{c}, -\frac{b}{a}\right)$ vertice

l'asse Ox $c>0$ ⊂

$c<0$ ⊃

Le equazioni parametriche

γ_1: $\begin{cases} x = t \\ y = \frac{1}{c}(at+b)^2 - \frac{1}{c}\cdot d \end{cases}$ $t\in\mathbb{R}$

γ_2: $\begin{cases} x = \frac{1}{c}(at+b)^2 - \frac{d}{c} \\ y = t \end{cases}$ $t\in\mathbb{R}$

$\boxed{1}$ Disegnare la curva
Scrivere le equazioni parametriche

1) $\gamma: x^2-4x+y-1=0$

$(x-2)^2-4+y-1=0$

$(x-2)^2=-y+5$

$P(2,5)$ Oy $\cap$

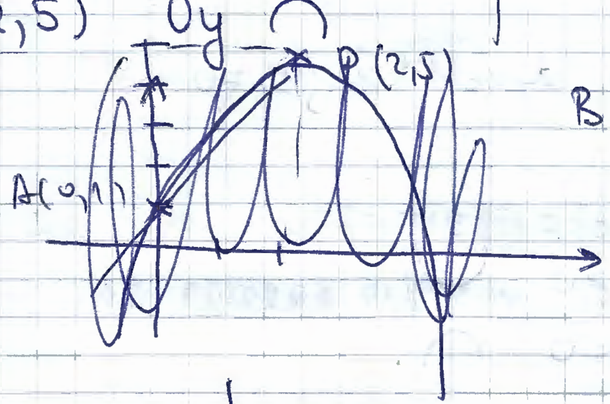
Punti di intersezione con gli assi

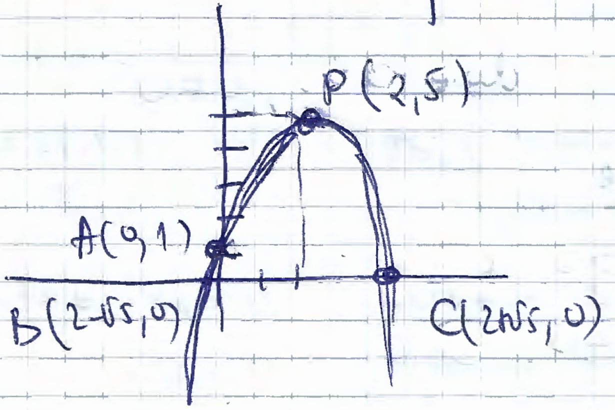
$Oy: x=0 \quad y=1 \quad A(0,1)$

$Ox: y=0 \quad x^2-4x-1=0$

$x_{1,2}=2\pm\sqrt{5}$

$B(2-\sqrt{5};0)$; $C(2+\sqrt{5};0)$





$x=t \Rightarrow t^2-4t+y-1=0 \quad y=-t^2+4t+1$

$\Rightarrow$ EP $\begin{cases} x=t \\ y=-t^2+4t+1 \end{cases}$ $t\in\mathbb{R}$

2) $\gamma: 2y^2+\frac{7}{2}y-\frac{3}{2}x+1=0$ $\quad$ $:2$

$\gamma: y^2+\frac{7}{4}y-\frac{3}{4}x+\frac{1}{2}=0$

$\left(y+\frac{7}{8}\right)^2-\frac{49}{64}-\frac{3}{2}x+\frac{1}{2}=0$ $\quad$ 32

$\left(y+\frac{7}{8}\right)^2=\frac{3}{2}x+\frac{17}{64}$

$y + \frac{7}{8} = 0 \quad y = -\frac{7}{8}$

$\frac{3}{2}x + \frac{17}{64} = 0 \quad x = -\frac{17}{64} : \frac{3}{2} = -\frac{17}{\cancel{64}_{32}} \cdot \frac{\cancel{2}^1}{3} = -\frac{17}{96}$

$\Rightarrow P\left(-\frac{17}{96}; -\frac{7}{8}\right) \quad Ox \quad \subset$

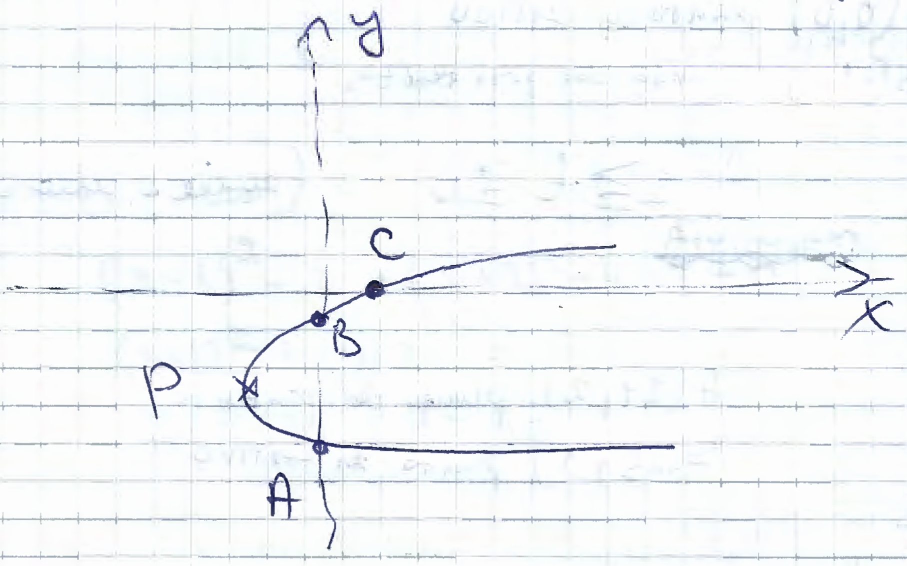
Punti di intersezione con gli assi

$x = 0 \quad 2y^2 + \frac{7}{2}y + 1 = 0 \quad 4y^2 + 7y + 2 = 0$

$\Delta = 49 - 32 = 17$

$y_{1,2} = \frac{-7 \pm \sqrt{17}}{8} \quad A\left(0, \frac{-7-\sqrt{17}}{8}\right) \quad B\left(0, \frac{-7+\sqrt{17}}{8}\right)$

$y = 0 \quad \frac{3}{2}x = 1 \quad x = \frac{2}{3} \quad C\left(\frac{2}{3}, 0\right)$



$\boxed{EP}$ $y = t \quad 2t^2 + \frac{7}{2}t - \frac{3}{2}x + 1 = 0$

$3x = 4t^2 + 7t + 2 \quad \begin{cases} x = \frac{4}{3}t^2 + \frac{7}{3}t + \frac{2}{3} \\ y = t \end{cases} \quad t \in R$

3) $x^2 + 5x - 2y = 0$

4) $2x^2 - 3x + y + 5 = 0$

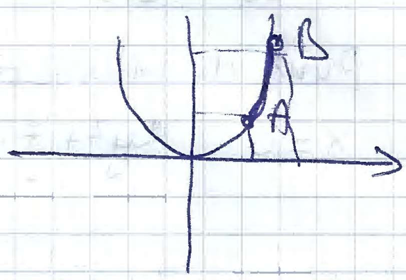
5) $y^2 - y + x = 0$

6) $3y^2 - 2y + 3x - 1 = 0$

$\boxed{2}$ Disegnare la curva

Scrivere le equazioni parametriche dell'arco AB della curva

1) $y = x^2$ $A(1,1)$ $B(2,4)$



$$\begin{cases} x = t \\ y = t^2 \end{cases}$$

$x = t$ $x_A = 1$ $t = 1$ $\Rightarrow t \in [1,2]$

$x_B = 2$ $t = 2$

2) $x = -y^2$ $A(0,0)$ punto di arrivo

$B(-1,1)$ punto di partenza

$$\begin{cases} x = -t^2 \\ y = t \end{cases}$$

$1 \geq t \geq 0$ (dalle 1 allo 0)

3) $x+1 = (y+3)^2$ $A(24,2)$ punto di partenza

$B(13,1)$ punto di arrivo

$$\begin{cases} x = (t+3)^2 - 1 \\ y = t \end{cases}$$

$2 \geq t \geq 1$

4) $y = -x^2$ $A(0,0)$ $B(2,-4)$

5) $x^2 - 2x + y + 1 = 0$ $A(1,0)$ p. di partenza

$B(2,-1)$ p. di arrivo

6) $y^2 + y + x - 2 = 0$ $A(2,0)$ p. di partenza

$B(0,1)$ p. di arrivo

III La circonferenza

$$\gamma: (x-a)^2 + (y-b)^2 = R^2$$

con il centro $C(a, b)$ e il raggio R

le Equazioni parametriche

$$\begin{cases} x = a + R\cos t \\ y = b + R\,\mathrm{sen}\,t \end{cases} \quad 0 \leq t \leq 2\pi$$

(più precisato $0 \leq t < 2\pi$)

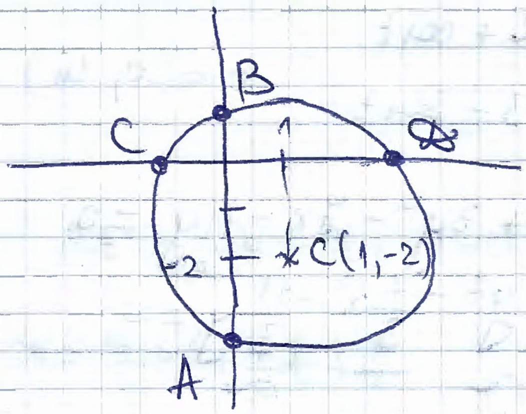
$\boxed{1}$ Disegnare la curva
Scrivere le equazioni parametriche

1) $\gamma: x^2 + y^2 - 2x + 4y - 1 = 0$

$(x-1)^2 - 1 + (y+2)^2 - 4 - 1 = 0$

$(x-1)^2 + (y+2)^2 = 6$

$C(1, -2) \quad R = \sqrt{6}$



Intersezione con gli assi

$x=0 \quad y^2 + 4y - 1 = 0$

$y_{1,2} = -2 \pm \sqrt{5}$

$A(0, -2-\sqrt{5}) \quad B(0, -2+\sqrt{5})$

$y=0 \quad x^2 - 2x - 1 = 0$

$x_{1,2} = 1 \pm \sqrt{2}$

$C(1-\sqrt{2}; 0) \quad D(1+\sqrt{2}; 0)$

EP $\begin{cases} x = 1 + \sqrt{6}\cos t \\ y = -2 + \sqrt{6}\,\mathrm{sen}\,t \end{cases} \quad t \in [0, 2\pi)$

2) $\gamma: x^2 + y^2 - 4x + 6y + 12 = 0$ $\qquad = 32 =$

$(x-2)^2 - 4 + (y+3)^2 - 9 + 12 = 0$

$(x-2)^2 + (y+3)^2 = 1$

$C(2,-3) \quad R = 1$

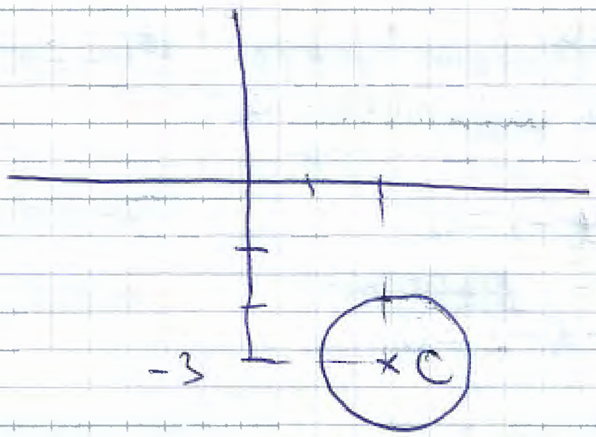
Intersezione con gli assi

$x = 0 \quad y^2 + 6y + 12 = 0$

$\Delta = 9 - 12 < 0$ non ci sono i radici (non ci sono i punti di intersezione)

$y = 0 \quad \gamma: x^2 - 4x + 12 = 0$

$\Delta = 4 - 12 < 0$ non ci sono i radici



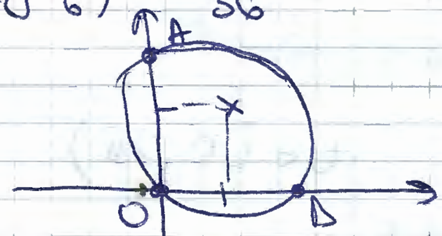
$$\begin{cases} x = 2 + \cos t \\ y = -3 + \operatorname{sen} t \end{cases} \quad t \in [0, 2\pi)$$

3) $\gamma: 3x^2 + 3y^2 - 7x - 11y = 0 \quad |:3$

$x^2 + y^2 - \frac{7}{3}x - \frac{11}{3}y = 0$

$\left(x - \frac{7}{6}\right)^2 - \frac{49}{36} + \left(y - \frac{11}{6}\right)^2 - \frac{121}{36} = 0$

$\left(x - \frac{7}{6}\right)^2 + \left(y - \frac{11}{6}\right)^2 = \frac{170}{36} \qquad C\left(\frac{7}{6}; \frac{11}{6}\right) \quad R = \frac{\sqrt{170}}{6}$



$x = 0 \quad 3y^2 - 11y = 0 \quad y(3y - 11) = 0 \quad O(0,0) \quad A\left(0, \frac{11}{3}\right)$

$y = 0 \quad 3x^2 - 7x = 0 \quad x(3x - 7) = 0 \quad O(0,0) \quad B\left(\frac{7}{3}, 0\right)$

EP $\begin{cases} x = \frac{7}{6} + \frac{\sqrt{170}}{6}\cos t \\ y = \frac{11}{6} + \frac{\sqrt{170}}{6}\operatorname{sen} t \end{cases}$ $\quad 0 \leq t \leq 2\pi$

4) γ: $x^2+y^2-4x+y=0$

5) γ: $x^2+y^2+x-y=0$

6) γ: $3x^2+3y^2-x+y-1=0$

7) γ: $7x^2+7y^2+5x+7y-5=0$

$\boxed{2}$ Disegnare la curva

Scrivere le equazioni parametriche dell'arco AB della curva a) in senso orario

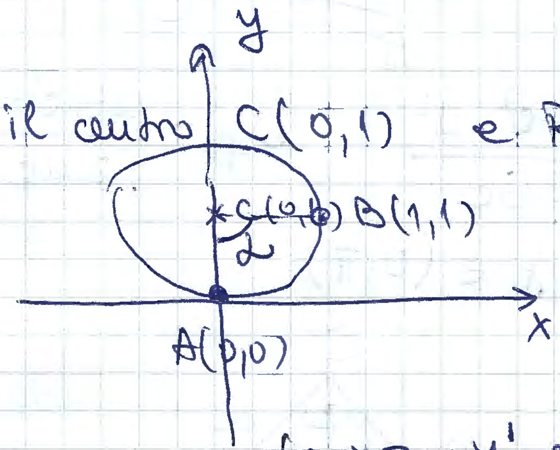
b) in senso antiorario

1) γ: $x^2+y^2-2y=0$

A(0,0) B(1,1)

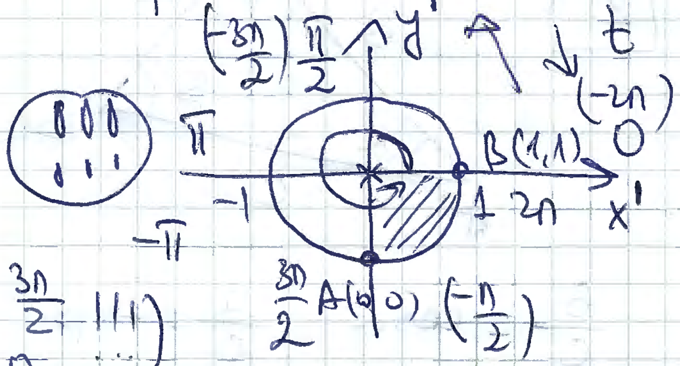
$x^2+(y-1)^2-1=0$ il centro C(0,1) e R=1

$x^2+(y-1)^2=1$



$\measuredangle\alpha = \frac{\pi}{2}$

$\begin{cases} x = \cos t \\ y = 1+\operatorname{sen} t \end{cases}$



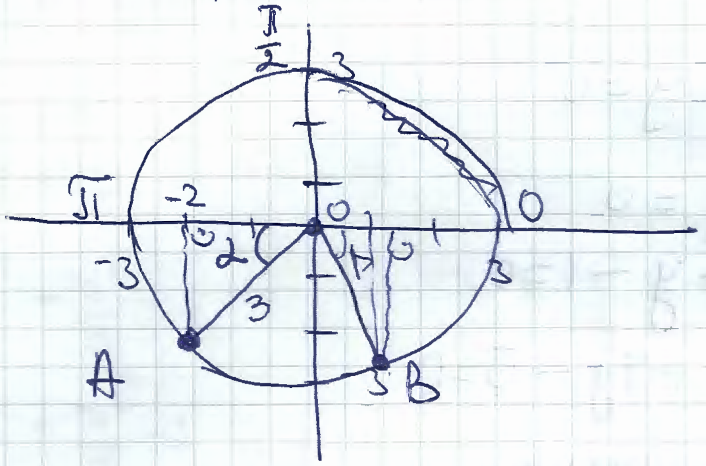
a) $\frac{3\pi}{2} \geq t \geq 0$ $\left(\begin{matrix}\text{da} & \frac{3\pi}{2} & !!!\\ \text{a} & 0 & \end{matrix}\right)$

b) $\frac{3\pi}{2} \leq t \leq 2\pi$ o $-\frac{\pi}{2} \leq t \leq 0$

2) $x^2+y^2=9$ =34=

$A(-2,-\sqrt{5})$ $B(1,-2\sqrt{2})$

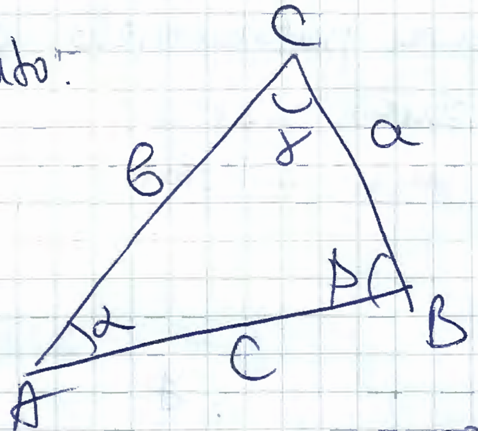
il centro $O(0,0)$ $R=3$



$\begin{cases} x=3\cos t \\ y=3\operatorname{sen} t \end{cases}$

Suggerimento:

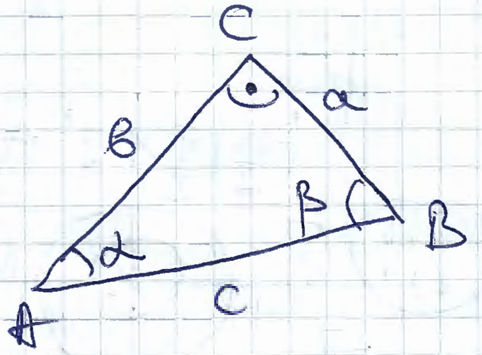
1



$\cos\alpha = \dfrac{b^2+c^2-a^2}{2bc}$ $\cos\beta = \dfrac{a^2+c^2-b^2}{2ac}$ $\cos\gamma = \dfrac{a^2+b^2-c^2}{2ab}$

$\alpha, \beta, \gamma \in (0,\pi)$

2



$\dfrac{b}{c} = \cos\alpha$ $\dfrac{a}{c} = \cos\beta$ $\underline{\alpha,\beta \in \left(0,\frac{\pi}{2}\right)}$!!!

$\dfrac{b}{c} = \operatorname{sen}\beta$ $\dfrac{a}{c} = \operatorname{sen}\alpha$

a) $\cos\alpha = \frac{2}{3} \Rightarrow \alpha = \arccos\frac{2}{3} \in \left(0, \frac{\pi}{2}\right)$

$\cos\beta = \frac{1}{3} \Rightarrow \beta = \arccos\frac{1}{3} \in \left(0, \frac{\pi}{2}\right)$

$$\begin{cases} x = 3\cos t \\ y = 3\,\mathrm{sen}\,t \end{cases}$$

$\pi + \arccos\frac{2}{3} \geq t \geq -\arccos\frac{1}{3}$

b) $\pi + \arccos\frac{2}{3} \leq t \leq 2\pi - \arccos\frac{1}{3}$

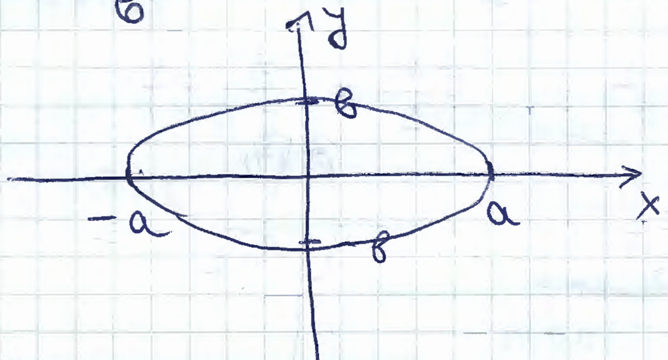
3) $x^2 + y^2 + 2x = 0$ $A(0,0)$; $B(-1,1)$

4) $x^2 + y^2 = 4$ $A(\sqrt{3}, 1)$; $B(1; -\sqrt{3})$

5) $x^2 + y^2 - 6x - 4y - 3 = 0$ $A(1; 2+2\sqrt{3})$; $B(0; 2-\sqrt{7})$

IV L'ellisse

$\gamma: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a > 0$ $b > 0$



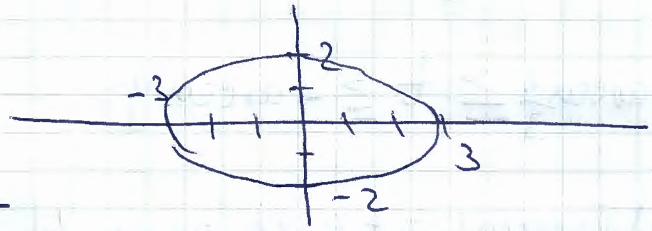
Le equazioni parametriche

$$\begin{cases} x = a\cos t \\ y = b\,\mathrm{sen}\,t \end{cases} \quad 0 \leq t \leq 2\pi \qquad (0 \leq t < 2\pi)$$

=36=

$\boxed{1}$ Disegnare la curva
Scrivere le equazioni parametriche

1) $\frac{x^2}{9} + \frac{y^2}{4} = 1$



$\begin{cases} x = 3\cos t \\ y = 2\operatorname{sen} t \end{cases}$ $0 \le t < 2\pi$

2) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ 3) $x^2 + \frac{y^2}{3} = 1$

4) $\frac{x^2}{2} + \frac{y^2}{7} = 1$

5) $2x^2 + 3y^2 = 6$ Sugg. $:6$

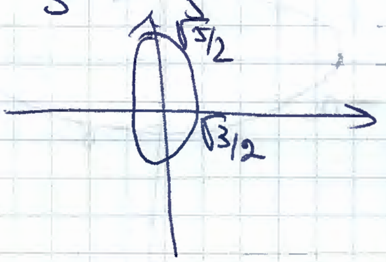
$\frac{2x^2}{6} + \frac{3y^2}{6} = 1$

$\frac{x^2}{3} + \frac{y^2}{2} = 1$

6) $10x^2 + 6y^2 = 15$ Sugg. $:15$

$\frac{10x^2}{15} + \frac{6y^2}{15} = 1$ $\frac{2x^2}{3} + \frac{2y^2}{5} = 1$

$\frac{x^2}{3/2} + \frac{y^2}{5/2} = 1$

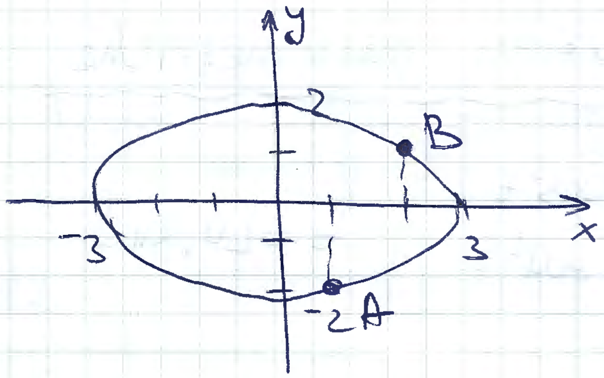


$\boxed{2}$ Disegnare la curva
Scrivere le equazioni parametriche dell'arco AB della curva a) in senso orario
b) in senso antiorario

1) $\gamma: 4x^2 + 9y^2 = 36$ $A\left(1, -\frac{4\sqrt{2}}{3}\right)$; $B\left(2, \frac{2\sqrt{5}}{3}\right)$

$$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$$

$$\gamma: \frac{x^2}{9} + \frac{y^2}{4} = 1$$



$$\begin{cases} x = 3\cos t \\ y = 2\operatorname{sen} t \end{cases}$$

punto $A\left(1, -\frac{4\sqrt{2}}{3}\right) \Rightarrow \begin{cases} 3\cos t = 1 \\ 2\operatorname{sen} t = -\frac{4\sqrt{2}}{3} \end{cases}$ $\begin{cases} \cos t = \frac{1}{3} \\ \operatorname{sen} t = -\frac{2\sqrt{2}}{3} \end{cases}$

$\Rightarrow t \in$ IV quadrante $\quad t = \arccos\frac{1}{3} \in \left(0, \frac{\pi}{2}\right)$

punto $B\left(2, \frac{2\sqrt{5}}{3}\right) \Rightarrow \begin{cases} 3\cos t = 2 \\ 2\operatorname{sen} t = \frac{2\sqrt{5}}{3} \end{cases}$ $\begin{cases} \cos t = \frac{2}{3} \\ \operatorname{sen} t = \frac{\sqrt{5}}{3} \end{cases}$

$\Rightarrow t \in$ I quadrante $\quad t = \arccos\frac{2}{3} \in \left(0, \frac{\pi}{2}\right)$

a) $2\pi - \arccos\frac{1}{3} \geq t \geq \arccos\frac{2}{3}$

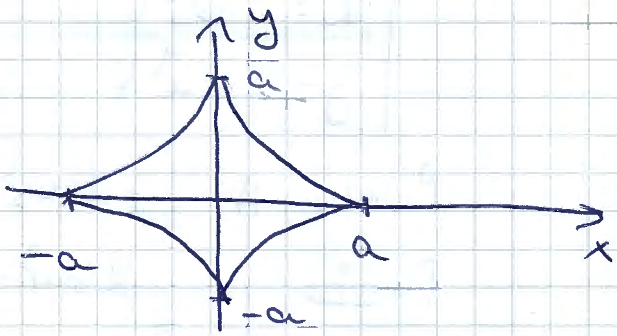
b) $-\arccos\frac{1}{3} \leq t \leq \arccos\frac{2}{3}$

2) $\gamma: 9x^2 + 4y^2 = 36$ $\quad A\left(\frac{4\sqrt{2}}{3}, 1\right)$ $\quad B\left(1; \frac{3\sqrt{3}}{2}\right)$

3) $\gamma: x^2 + 4y = 4$ $\quad A\left(1; -\frac{\sqrt{3}}{2}\right)$ $\quad B\left(\sqrt{3}; \frac{1}{2}\right)$

V L'astroide

$$x^{2/3} + y^{2/3} = a^{2/3} \qquad a > 0$$



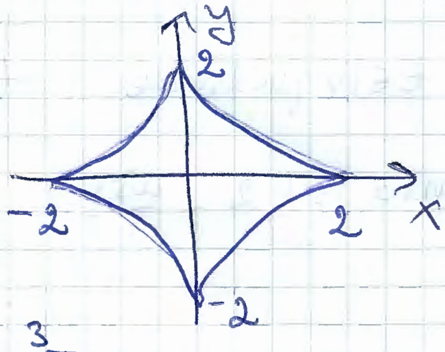
Le equazioni parametriche

$$\begin{cases} x = a\cos^3 t \\ y = a\,\text{sen}^3 t \end{cases} \quad t \in [0, 2\pi] \quad \left(t \in [0, 2\pi) \right)$$

$\boxed{1}$ Disegnare la curva. Scrivere le equazioni parametriche

1) $x^{2/3} + y^{2/3} = 2^{2/3}$

$$\begin{cases} x = 2\cos^3 t \\ y = 2\,\text{sen}^3 t \end{cases} \quad t \in [0, 2\pi)$$



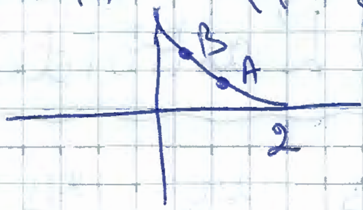
2) $x^{2/3} + y^{2/3} = 1$ 3) $x^{2/3} + y^{2/3} = \sqrt[3]{9}$

4) $x^{2/3} + y^{2/3} = \dfrac{1}{\sqrt[3]{4}}$

$\boxed{2}$ Disegnare la curva. Scrivere le equazioni parametriche dell'arco AB della curva

A punto di partenza
B punto di arrivo
in senso antiorario

1) $\gamma: x^{2/3} + y^{2/3} = 2^{2/3}$ $A\left(\frac{3\sqrt{3}}{4}, \frac{1}{4}\right)$ $B\left(\frac{1}{4}, \frac{3\sqrt{3}}{4}\right)$



$$\begin{cases} x = 2\cos^3 t \\ y = 2\,\text{sen}^3 t \end{cases}$$

Punto A $\quad 2\cos^3 t = \frac{3\sqrt{3}}{4} \quad \cos^3 t = \frac{3\sqrt{3}}{8} = \left(\frac{\sqrt{3}}{2}\right)^3 \quad \cos t = \frac{\sqrt{3}}{2}$

~~Punto~~ $\quad 2\,\text{sen}^3 t = \frac{1}{4} \quad \text{sen}^3 t = \frac{1}{8} \quad \text{sen}\, t = \frac{1}{2}$

$\Rightarrow t = \frac{\pi}{6}$

Punto B $\quad 2\cos^3 t = \frac{1}{4} \quad \cos t = \frac{1}{2}$

$2\,\text{sen}^3 t = \frac{3\sqrt{3}}{4} \quad \text{sen}\, t = \frac{\sqrt{3}}{2}$ $\Big\} \Rightarrow t = \frac{\pi}{3}$

$\Rightarrow$ AB: $\begin{cases} x = 2\cos^3 t \\ y = 2\,\text{sen}^3 t \end{cases}$

$\frac{\pi}{6} \le t \le \frac{\pi}{3}$

2) γ: $x^{2/3} + y^{2/3} = 3^{2/3}$ $\quad A\left(\frac{3\sqrt{2}}{4}, -\frac{3\sqrt{2}}{4}\right) \quad B\left(\frac{9\sqrt{3}}{8}, \frac{3}{8}\right)$

3) γ: $x^{2/3} + y^{2/3} = 1$ $\quad A\left(\frac{3\sqrt{3}}{8}, \frac{1}{8}\right) \quad B\left(-\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4}\right)$

VI L'iperbole

Suggerimento:

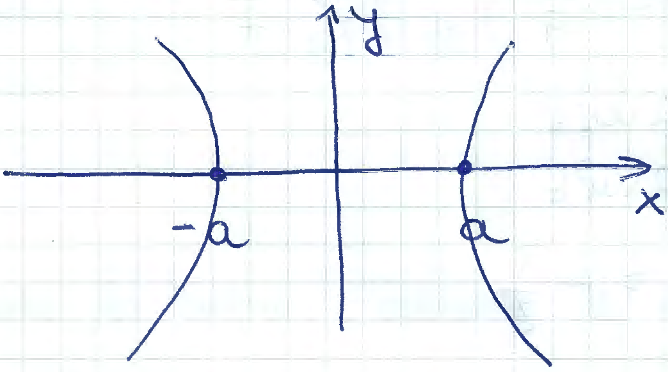
1) $\text{senh}\, t = \frac{e^t - e^{-t}}{2} \qquad \cosh t = \frac{e^t + e^{-t}}{2}$

2) $\cosh^2 t - \text{senh}^2 t = 1$

3) $(\cosh t)' = \text{senh}\, t$

$(\text{senh}\, t)' = \cosh t$

$\boxed{1}$ $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \qquad a>0 \quad b>0$

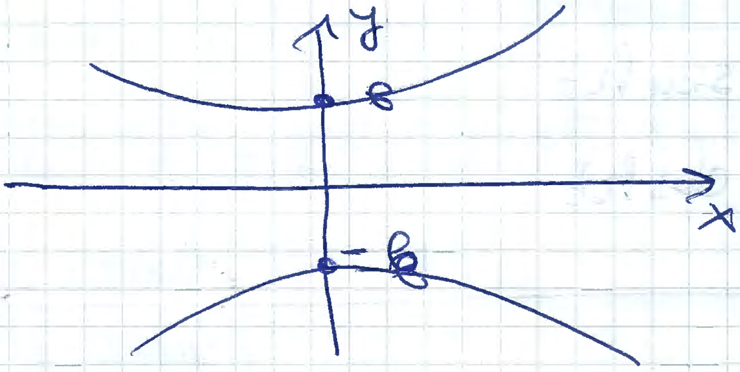


Le equazioni parametriche

$x>0 \quad \begin{cases} x = a\cosh t \\ y = b \operatorname{senh} t \\ t \in \mathbb{R} \end{cases}$

$x<0 \quad \begin{cases} x = -a\cosh t \\ y = b \operatorname{senh} t \\ t \in \mathbb{R} \end{cases}$

$\boxed{2}$ $-\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad a>0 \quad b>0$



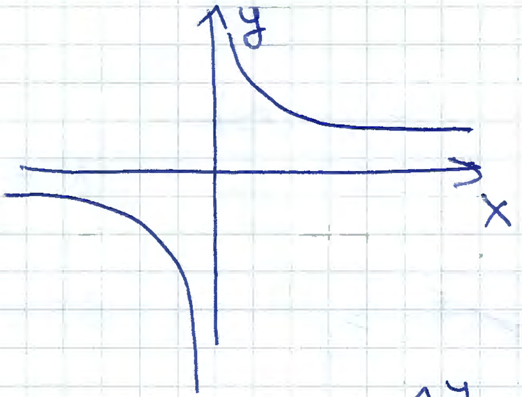
Le equazioni parametriche

$y>0 \quad \begin{cases} x = a\,\mathrm{senh}\,t \\ y = b\cosh t \\ t \in \mathbb{R} \end{cases}$

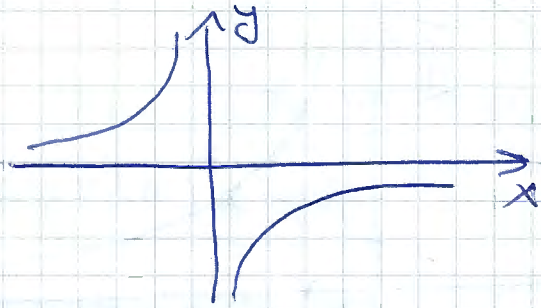
$y<0 \quad \begin{cases} x = a\,\mathrm{senh}\,t \\ y = -b\cosh t \\ t \in \mathbb{R} \end{cases}$

$\boxed{3}$ $\quad x \cdot y = a \neq 0$

$a>0$



$a<0$



Le equazioni parametriche

$\begin{cases} x = t \\ y = \frac{a}{t} \\ t \in (-\infty; 0) \cup (0; +\infty) \end{cases}$ o $\begin{cases} x = \frac{a}{t} \\ y = t \\ t \in (-\infty; 0) \cup (0; +\infty) \end{cases}$