Richiami di calcolo vettoriale e matriciale

(I) Determinanti

(1) Il determinante di ordine 2

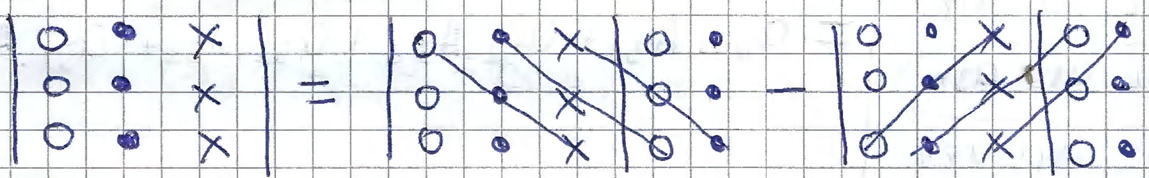
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - b \cdot c$$

1) $\begin{vmatrix} 2 & -3 \\ -4 & -7 \end{vmatrix} = 2 \cdot (-7) - (-3) \cdot (-4) = -14 - 12 = -26$

2) $\begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = \cos^2 x + \operatorname{sen}^2 x = 1$

3) $\begin{vmatrix} x+1 & -1 \\ x^2 & -3x \end{vmatrix} = (x+1) \cdot (-3x) - (-1) \cdot x^2 = -3x^2 - 3x + x^2 = -2x^2 - 3x$

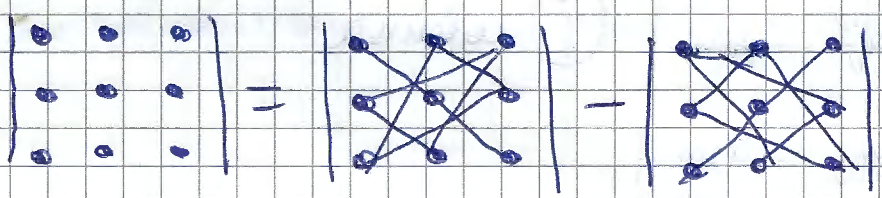
(2) Il determinante di ordine 3



(La regola di Sarrus)

1) $\left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ -1 & -2 & -5 & -1 & -2 \end{array}\right|$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & -2 & -5 \end{vmatrix} = 1 \cdot 5 \cdot (-5) + (2) \cdot 6 \cdot (-1) + 3 \cdot 4 \cdot (-2) - [2 \cdot 4 \cdot (-5) +$$
$$+ 1 \cdot 6 \cdot (-2) + 3 \cdot 5 \cdot (-1)] =$$
$$= -25 - 12 - 24 - (-40 - 12 - 15) = -25 - \cancel{12} - 24 + 40 \cancel{+12} + 15 = -49 + 55 = 6$$



2) $\begin{vmatrix} \cos x & \operatorname{sen} x & x \\ -\operatorname{sen} x & \cos x & 1 \\ -\cos x & -\operatorname{sen} x & 0 \end{vmatrix} = \cos x \cdot \cos x \cdot 0 + (-\operatorname{sen} x) \cdot (-\operatorname{sen} x) \cdot x +$

$+ (\operatorname{sen} x) \cdot 1 \cdot (-\cos x) - [x \cdot \cos x \cdot (-\cos x) + 1 \cdot (-\operatorname{sen} x) \cdot \cos x +$

$+ \operatorname{sen} x \cdot (-\operatorname{sen} x) \cdot 0] =$

$= 0 + x \operatorname{sen}^2 x - \operatorname{sen} x \cos x - [-x \cdot \cos^2 x - \operatorname{sen} x \cos x + 0] =$

$= x \cdot \operatorname{sen}^2 x - \cancel{\operatorname{sen} x \cos x} + x \cdot \cos^2 x + \cancel{\operatorname{sen} x \cos x} =$

$= x \cdot [\operatorname{sen}^2 x + \cos^2 x] = x$

(3) Il determinante di ordine ≥ 4

Regola di Laplace

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{31} \cdot A_{31} + a_{32} \cdot A_{32} + a_{33} \cdot A_{33} + a_{34} \cdot A_{34}$$

è sviluppato secondo la terza riga

è indifferente sviluppare secondo una qualsiasi riga o colonna.

A_{ij} complemento algebrico di a_{ij}

$A_{ij} = (-1)^{i+j} \cdot \Delta_{ij}$

Δ_{ij} complemento di a_{ij}

$$\Delta_{ij} = \begin{vmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ a_{i1} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & & & & \\ a_{n1} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix}$$

cancellata (i) riga e (j) colonna

1) $$\Delta = \begin{vmatrix} tg\,x & sen\,x & x & 1 \\ \cos x & \cos x & 1 & 0 \\ -sen\,x & -sen\,x & 0 & 1 \\ x & x^2 & -1 & 0 \end{vmatrix} = 1 \cdot A_{14} + 0 \cdot A_{24} + 1 \cdot A_{34} + 0 \cdot A_{44} =$$

$$= A_{14} + A_{34}$$

$$A_{14} = (-1)^{1+4} \cdot \Delta_{14} = -\Delta_{14}$$

$$\Delta_{14} = \begin{vmatrix} \cos x & \cos x & 1 \\ -sen\,x & -sen\,x & 0 \\ x & x^2 & -1 \end{vmatrix} = \cancel{sen\,x \cos x} - x^2 sen\,x + x\,sen\,x - \cancel{sen\,x \cos x}$$
$$= (-x^2 + x)\, sen\,x$$

$$\boxed{A_{14} = (x^2 - x)\, sen\,x}$$

$$A_{34} = (-1)^{3+4} \cdot \Delta_{34} = -\Delta_{34}$$

$$\Delta_{34} = \begin{vmatrix} tg\,x & sen\,x & x \\ \cos x & \cos x & 1 \\ x & x^2 & -1 \end{vmatrix} = -tg\,x \cdot \cos x + x^3 \cos x + x\,sen\,x -$$
$$- x^2 \cos x - x^2\, tg\,x + sen\,x \cos x$$

$$A_{34} = \underline{\sin x} - \underline{\underline{x^3 \cos x}} - \underline{x\, sen\,x} + \underline{\underline{x^2 \cos x}} + x^2\, tg\,x - sen\,x \cos x =$$

$$\boxed{= (1-x)\, sen\,x + (-x^3 + x^2) \cos x + x^2\, tg\,x - sen\,x \cos x}$$

$$\Delta = (x^2 - x)\, sen\,x + (1-x)\, sen\,x + (x^2 - x^3) \cos x + x^2\, tg\,x - sen\,x \cos x$$

$$\Delta = (x^2 - 2x + 1)\, sen\,x + (x^2 - x^3) \cos x + x^2\, tg\,x - sen\,x \cos x$$

4 Calcolare:

1) $\begin{vmatrix} 2 & -1 \\ -3 & -4 \end{vmatrix}$ 2) $\begin{vmatrix} -1 & -2 \\ 3 & -5 \end{vmatrix}$ 3) $\begin{vmatrix} 0 & -1 \\ 2 & -3 \end{vmatrix}$ 4) $\begin{vmatrix} 3 & 0 \\ -4 & -2 \end{vmatrix}$

5) $\begin{vmatrix} x & y \\ z & t \end{vmatrix}$ 6) $\begin{vmatrix} 1 & -1 & 2 \\ 0 & -1 & -3 \\ -4 & 2 & -3 \end{vmatrix}$ 7) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & -1 & 2 \\ 3 & 0 & -4 \end{vmatrix}$

8) $\begin{vmatrix} 3 & -1 & 1 \\ 4 & -1 & 3 \\ 5 & -2 & -1 \end{vmatrix}$ 9) $\begin{vmatrix} \operatorname{tg} x & \operatorname{sen} x \\ \frac{1}{\cos^2 x} & \cos x \end{vmatrix}$

10) $\begin{vmatrix} x & -x^2 & 1 \\ 2x & -x & -3 \\ 0 & 3x^3 & 5x \end{vmatrix}$ 11) $\begin{vmatrix} \sin^2 x & \cos^2 x & x \\ \sin 2x & -\sin 2x & 1 \\ 2\cos 2x & -2\cos 2x & 0 \end{vmatrix}$

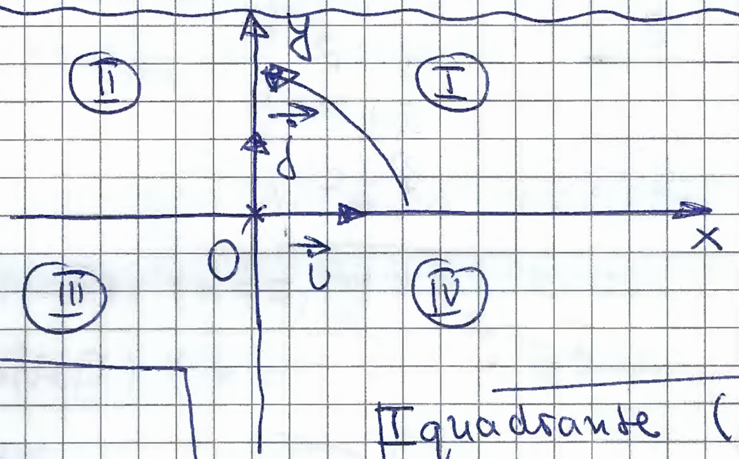
12) $\begin{vmatrix} 1 & -1 & 2 & 0 \\ -3 & 0 & -4 & 2 \\ 3 & -1 & 2 & 5 \\ 0 & 3 & -1 & 0 \end{vmatrix}$ 13) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

14) $\begin{vmatrix} \sin x & \cos x & x & 0 \\ -\cos x & \sin x & -1 & x \\ \sin x & \cos x & 0 & x^2 \\ \cos x & -\sin x & 0 & -3 \end{vmatrix}$

II Sistemi di riferimento cartesiani

1 Sistema di riferimento ortonormale standard (destroso)

R2



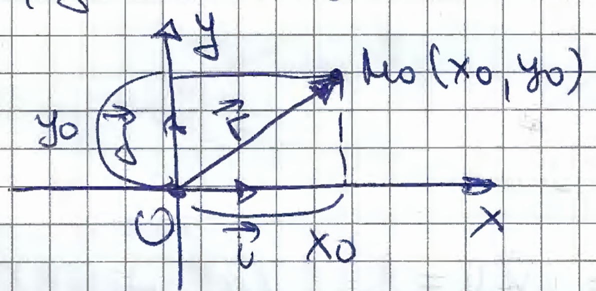
[1]
1) $Ox \perp Oy$
2) $\|\vec{i}\| = \|\vec{j}\| = 1$
3) $(\vec{i}, \vec{j})$ destroso

[4]
I quadrante $(x>0, y>0)$
II quadrante $(x<0, y>0)$
III quadrante $(x<0, y<0)$
IV quadrante $(x>0, y<0)$

[2] p. O origine

[3]
Ox l'asse delle ascisse o l'assissa
Oy l'asse delle ordinate o l'ordinata

[5] $\vec{i}, \vec{j}$ versori fondamentali (canonici)



[6] punto $M_0(x_0, y_0)$

[7] $\overrightarrow{OM}$ raggio vettore di p. M_0 $\quad \overrightarrow{OM}(x_0, y_0) = \vec{r}$

[8] $\vec{a}(a_1, a_2) = a_1 \cdot \vec{i} + a_2 \vec{j}$

$\vec{a}(2, -3) \Rightarrow \vec{a} = 2\vec{i} - 3\vec{j}$

$\vec{b} = -3\vec{j} - 2\vec{i} \Rightarrow \vec{b}(-2, -3)$

[9] $\vec{i}(1, 0)$; $\vec{j}(0, 1)$; $O(0, 0)$

[10] Sia $x_0, y_0 \neq 0$

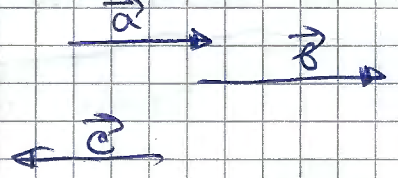
$\vec{a}(x_0, 0) \begin{cases} \parallel Ox \\ \perp Oy \end{cases}$ $\qquad \vec{b}(0, y_0) \begin{cases} \parallel Oy \\ \perp Ox \end{cases}$

$\vec{0}(0, 0)$ vettore nullo

[11] $A(x_1, y_1)$ $B(x_2, y_2)$ $\Rightarrow \overrightarrow{AB}(x_2 - x_1, y_2 - y_1)$

$A(-3, 1)$ $B(2, -4)$ $\Rightarrow \overrightarrow{AB}(5, -5)$

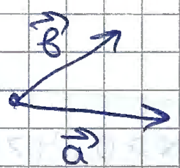
[12]



$\vec{a} \parallel \vec{b}$

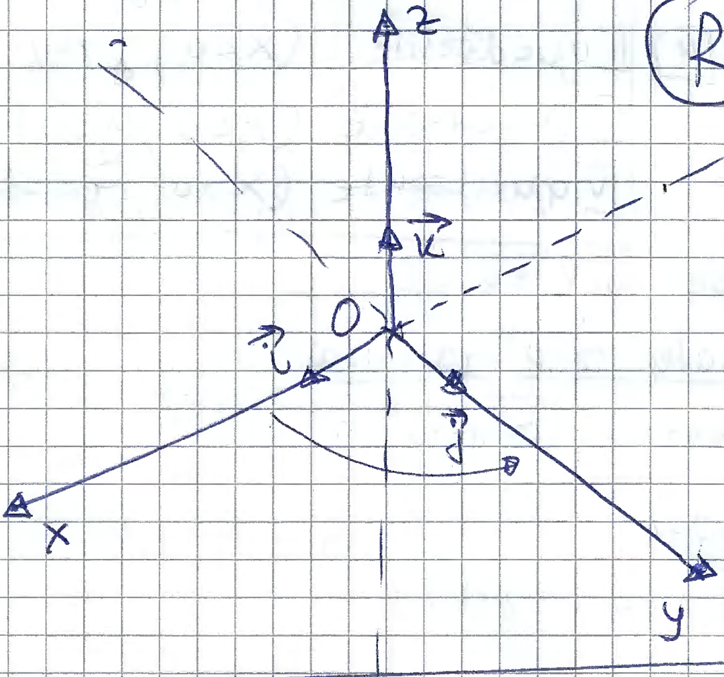
$\vec{a} \parallel \vec{c}$

$\vec{b} \parallel \vec{c}$



$\vec{a} \nparallel \vec{b}$

R 3



[1]

1) $Ox \perp Oy$, $Ox \perp Oz$, $Oy \perp Oz$

2) $\|\vec{i}\| = \|\vec{j}\| = \|\vec{k}\| = 1$

3) $(\vec{i}, \vec{j}, \vec{k})$ destroso (mano destra)

[2] p. O origine

[3] L'asse Ox, l'asse Oy; l'asse Oz

[4] Piani

(Oxy) o (xy) $\qquad$ $Oxy: z = 0$

(Oxz) o (xz) $\qquad$ $Oxz: y = 0$

(Oyz) o (yz) $\qquad$ $Oyz: x = 0$

5 Ottanti

I ottante $(x>0,\ y>0,\ z>0)$

II ottante $(x<0,\ y>0,\ z>0)$

III ottante $(x<0,\ y<0,\ z>0)$

IV ottante $(x>0,\ y<0,\ z>0)$

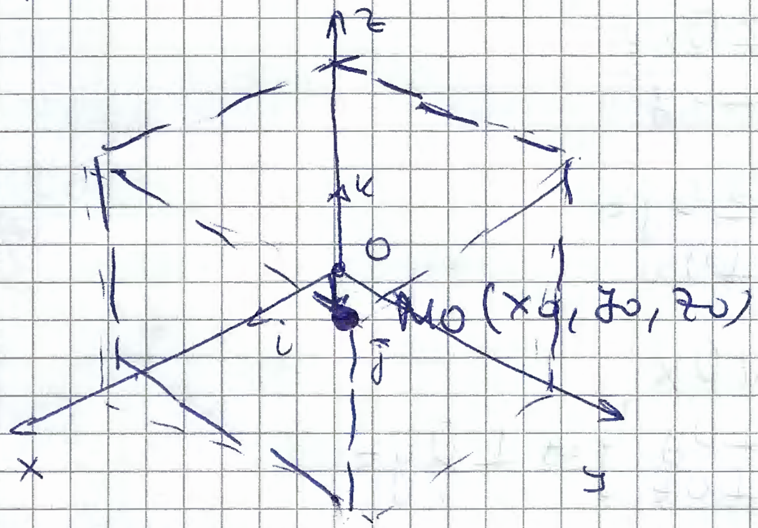
V ottante (sotto I) $(x>0,\ y>0,\ z<0)$

VI ottante (sotto II) $(x<0,\ y>0,\ z<0)$

VII ottante (sotto III) $(x<0,\ y<0,\ z<0)$

VIII ottante (sotto IV) $(x>0,\ y<0,\ z<0)$

6 $\vec{i}, \vec{j}, \vec{k}$ versori fundamentali (canonici)



$M_0(x_0, y_0, z_0)$

7 $\vec{OM} = \vec{r} = (x_0, y_0, z_0)$ raggio vettore di p. M_0

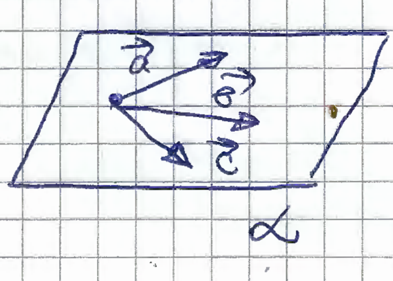
8 $\vec{a}(a_1, a_2, a_3) = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$\vec{a}(-1, 2, 0) \Rightarrow \vec{a} = -i + 2\vec{j}$

$\vec{b} = 3\vec{k} - 2\vec{i} - 5\vec{j} \Rightarrow \vec{b}(-2, -5, 3)$

9 $\vec{i}(1,0,0)$ $\vec{j}(0,1,0)$ $\vec{k}(0,0,1)$

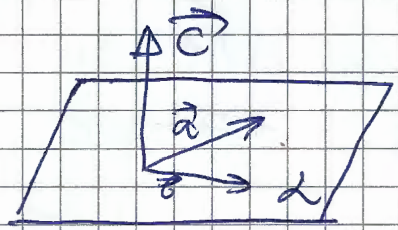
10



$\vec{a}, \vec{b}, \vec{c}$ sono complanari

$(\vec{a}, \vec{b}, \vec{c}) \supset \alpha$

"passa per"



$\vec{a}, \vec{b}, \vec{c}$ non sono complanari

$(\vec{a}, \vec{b}, \vec{c}) \not\subset \alpha$

Due vettori sono sempre complanari

Tre e più vettori possono essere complanari, possono essere non complanari.

$\boxed{11}$ Sia $x_0, y_0, z_0 \neq 0$

1) $\vec{a}(x_0, y_0, 0)$ | $\subset Oxy$ $(a \parallel Oxy)$; $\perp Oz$

2) $\vec{a}(x_0, 0, z_0)$ | $\subset Oxz$; $\perp Oy$

3) $\vec{a}(0, y_0, z_0)$ | $\subset Oyz$; $\perp Ox$

4) $\vec{a}(x_0, 0, 0)$ | $\parallel Ox$; $\left.\begin{array}{l}\perp Oy \\ \perp Oz\end{array}\right\} \Rightarrow \perp Oyz$

5) $\vec{a}(0, y_0, 0)$ | $\parallel Oy$; $\left.\begin{array}{l}\perp Ox \\ \perp Oz\end{array}\right\} \Rightarrow \perp Oxz$

6) $\vec{a}(0, 0, z_0)$ | $\parallel Oz$; $\left.\begin{array}{l}\perp Ox \\ \perp Oy\end{array}\right\} \Rightarrow \perp Oxy$

7) $\vec{0}(0,0,0)$ vettore nullo

$\boxed{12}$ $A(x_1, y_1, z_1)$; $B(x_2, y_2, z_2)$ $\Rightarrow \overrightarrow{AB}(x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$A(2, -3, 0)$ $\quad B(-1, 0, 4)$ $\Rightarrow \overrightarrow{AB}(-3, 3, 4)$

III Prodotto scalare – $\boxed{\text{numero}}$

① $\vec{a} \cdot \vec{b} = \|\vec{a}\| . \|\vec{b}\| . \cos \measuredangle(\vec{a}, \vec{b}) \in \mathbb{R}$

$0 \leq \measuredangle(\vec{a}, \vec{b}) \leq \pi \quad (0° \leq \measuredangle(\vec{a}, \vec{b}) \leq 180°)$

② $\cos \measuredangle(\vec{a}, \vec{b}) = \dfrac{\vec{a} . \vec{b}}{\|\vec{a}\| . \|\vec{b}\|}$

③ $\vec{a} . \vec{a} = \vec{a}^2 = \|\vec{a}\| . \|\vec{a}\| . \cos 0 = \|\vec{a}\|^2$

④ $\|\vec{a}\| = \sqrt{\vec{a}^2}$

⑤ Se $\vec{a} \perp \vec{b} \Rightarrow \vec{a} . \vec{b} = 0$

⑥ se $\vec{a} . \vec{b} = 0 \Rightarrow \begin{cases} \vec{a} \perp \vec{b} \ (\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}) \\ \vec{a} = \vec{0} \text{ o } \vec{b} = \vec{0} \text{ o } \vec{a} = \vec{0} \text{ e } \vec{b} = \vec{0} \end{cases}$

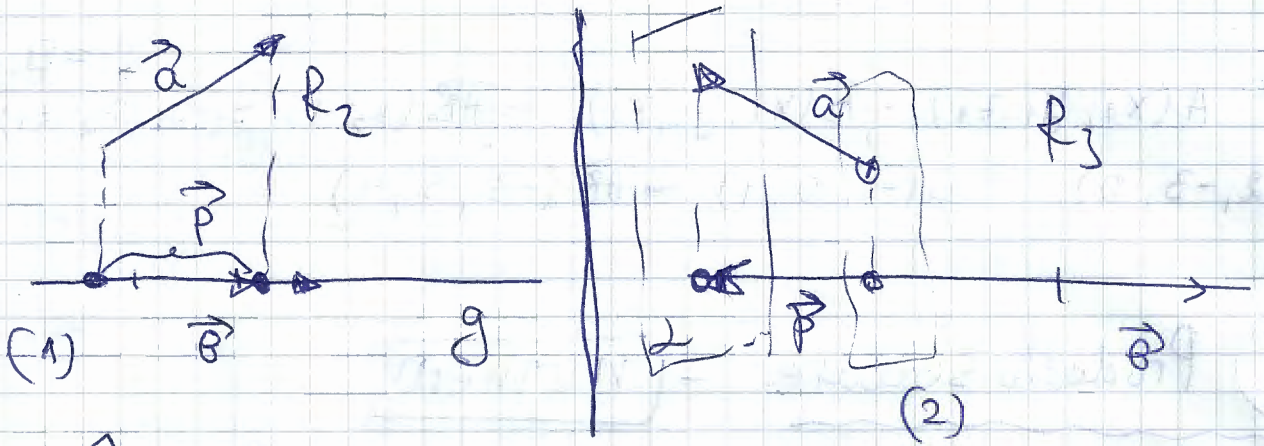
⑦ $\vec{a} . \vec{b} = \vec{b} . \vec{a}$

⑧ $(\vec{a} + \vec{b}) . \vec{c} = \vec{c}(\vec{a} + \vec{b}) = \vec{a}\vec{c} + \vec{b}\vec{c}$

⑨ !!! $(\vec{a} . \vec{b}) . \vec{c} \neq \vec{a} . (\vec{b} . \vec{c})$

⑩ $\hat{\vec{a}} = \dfrac{\vec{a}}{\|\vec{a}\|}$ vettore unitario (versore) di $\vec{a}$

⑪ Proiezione ortogonale di un vettore su un altro



1) $\hat{\vec{b}} = \dfrac{\vec{b}}{\|\vec{b}\|}$

2) $\vec{a} \cdot \hat{\vec{b}} = m \begin{cases} >0 & \text{se } \vec{b} \uparrow\uparrow \vec{p} \quad (1) \\ <0 & \text{se } \vec{b} \uparrow\downarrow \vec{p} \quad (2) \end{cases}$

$|m|$ è la lunghezza di $\vec{p}$

3) $\vec{p} = m \cdot \hat{\vec{b}} = \vec{pr}_{\vec{b}}\, \vec{a}$

$\boxed{R2}$

Sia $\vec{a}(a_1, a_2)$, $\vec{b}(b_1, b_2)$ $(\vec{i}(1,0); \vec{j}(0,1))$

① $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$

$\vec{a}(2,-3)$ $\vec{b}(-1,4)$ $\vec{a} \cdot \vec{b} = 2\cdot(-1) + (-3)\cdot 4 = -2-12 = -14$

② $\|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$

$\|\vec{a}\| = \sqrt{4+9} = \sqrt{13}$; $\|\vec{b}\| = \sqrt{1+16} = \sqrt{17}$

③ $\cos\sphericalangle(\vec{a}, \vec{b}) = \dfrac{\vec{a} \cdot \vec{b}}{\|a\| \cdot \|b\|}$

$\cos\sphericalangle(\vec{a}, \vec{b}) = \dfrac{-14}{\sqrt{13} \cdot \sqrt{17}}$

④ $\hat{\vec{a}} = \left(\dfrac{a_1}{\|\vec{a}\|}, \dfrac{a_2}{\|\vec{a}\|} \right)$

$\hat{\vec{a}} = \left(\dfrac{2}{\sqrt{13}}, \dfrac{-3}{\sqrt{13}} \right)$; $\hat{\vec{b}} = \left(-\dfrac{1}{\sqrt{17}}, \dfrac{4}{\sqrt{17}} \right)$

(5) $\cos \sphericalangle(\vec{a}, \vec{i}) = \dfrac{\vec{a} \cdot \vec{i}}{\|\vec{a}\| \|\vec{i}\|} = \dfrac{a_1}{\|\vec{a}\|} = \cos \alpha$

$\cos \sphericalangle(\vec{a}, \vec{j}) = \dfrac{\vec{a} \cdot \vec{j}}{\|\vec{a}\| \|\vec{j}\|} = \dfrac{a_2}{\|\vec{a}\|} = \cos \beta$

$\Rightarrow \hat{a}(\cos\alpha, \cos\beta) \Rightarrow \cos^2\alpha + \cos^2\beta = 1$

$\Rightarrow$ Se $a_1 > 0 \Rightarrow \sphericalangle(\vec{a}, \vec{i}) < \frac{\pi}{2}$ (acuto)

$a_1 = 0 \Rightarrow \sphericalangle(\vec{a}, \vec{i}) = \frac{\pi}{2}$ (retto)

$a_1 < 0 \Rightarrow \sphericalangle(\vec{a}, \vec{i}) > \frac{\pi}{2}$ (ottuso)

Se $a_2 > 0 \Rightarrow \sphericalangle(\vec{a}, \vec{j}) < \frac{\pi}{2}$ (acuto)

$a_2 = 0 \Rightarrow \sphericalangle(\vec{a}, \vec{j}) = \frac{\pi}{2}$ (retto)

$a_2 < 0 \Rightarrow \sphericalangle(\vec{a}, \vec{j}) > \frac{\pi}{2}$ (ottuso)

Esempio $\vec{a}(-3, 2) \Rightarrow \sphericalangle(\vec{a}, \vec{i})$ è ottuso

$\sphericalangle(\vec{a}, \vec{j})$ è acuto

È Dato: ~~vettori~~: $\vec{a}, \vec{b}, \vec{c}$ Trovate:

a) $\vec{a} \cdot \vec{b}$; $\vec{b} \cdot \vec{c}$;

b) $\|\vec{a}\|$; $\|\vec{b}\|$; $\|\vec{c}\|$

c) $\hat{a}$, $\hat{b}$, $\hat{c}$

d) $\sphericalangle(\vec{a}, \vec{b})$; $\sphericalangle(\vec{b}, \vec{c})$

e) $\left.\begin{matrix} \sphericalangle(\vec{a}, \vec{i}) ; \sphericalangle(\vec{a}, \vec{j}) \\ \sphericalangle(\vec{b}, \vec{i}) ; \sphericalangle(\vec{b}, \vec{j}) \\ \sphericalangle(\vec{c}, \vec{i}) ; \sphericalangle(\vec{c}, \vec{j}) \end{matrix}\right\}$ è acuto retto ottuso ?

f) $\overrightarrow{np}_{\vec{b}} \vec{a}$; $\overrightarrow{np}_{\vec{a}} \vec{b}$

(A) $\vec{a}(3, -1)$; $\vec{b}(-1, 2)$, $\vec{c}(-3, 0)$

f) $\hat{\vec{b}} = \frac{\vec{b}}{\|\vec{b}\|}$

1) $\|\vec{b}\| = \sqrt{1+4} = \sqrt{5}$

$\hat{\vec{b}} = \left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right)$

2) $m = \vec{a} \cdot \hat{\vec{b}} = \frac{-3}{\sqrt{5}} - \frac{2}{\sqrt{5}} = \frac{-5}{\sqrt{5}} = -\sqrt{5}$

3) $\overrightarrow{np}_{\vec{b}}\, \vec{a} = m \cdot \hat{\vec{b}} = \left(\frac{+\sqrt{5}}{\sqrt{5}}, \frac{-2\sqrt{5}}{\sqrt{5}} \right) = (1, -2)$

(B) $\vec{a}(2,-3)$ $\vec{b}(-4,3)$ $\vec{c}(-1,1)$

(C) $\vec{a}(-1,0)$ $\vec{b}(5,-1)$ $\vec{c}(2,2)$

(D) $\vec{a}(3,0)$ $\vec{b}(0,-2)$ $\vec{c}(3,-2)$

$\boxed{R3}$

Sia $\vec{a}(a_1,a_2,a_3)$; $\vec{b}(b_1,b_2,b_3)$

$(\vec{i}(1,0,0)$; $\vec{j}(0,1,0)$; $\vec{k}(0,0,1))$

(1) $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ $\quad \vec{a}(-1,3,0)$ $\vec{b}(2,-3,-7)$

$\vec{a} \cdot \vec{b} = (-1) \cdot 2 + 3 \cdot (-3) + 0 \cdot (-7) = -2 - 9 + 0 = -11$

(2) $\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$\|\vec{a}\| = \sqrt{1+4+0} = \sqrt{5}$ $\quad \|\vec{b}\| = \sqrt{4+9+49} = \sqrt{62}$

(3) $\cos \measuredangle(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$

$\cos \measuredangle(\vec{a}, \vec{b}) = \frac{-11}{\sqrt{5} \cdot \sqrt{62}}$

(4) $\hat{\vec{a}} = \left(\frac{a_1}{\|\vec{a}\|}, \frac{a_2}{\|\vec{a}\|}, \frac{a_3}{\|\vec{a}\|} \right)$

$\hat{\vec{a}} = \left(\frac{-1}{\sqrt{5}}, \frac{3}{\sqrt{5}}, \frac{0}{\sqrt{5}} = 0 \right) \qquad \hat{\vec{b}} = \left(\frac{2}{\sqrt{62}}, \frac{-3}{\sqrt{62}}, \frac{-7}{\sqrt{62}} \right)$

(5) $\cos \sphericalangle(\vec{a}, \vec{i}) = \frac{a_1}{\|\vec{a}\|} = \cos\alpha$

$\cos \sphericalangle(\vec{a}, \vec{j}) = \frac{a_2}{\|\vec{a}\|} = \cos\beta$

$\cos \sphericalangle(\vec{a}, \vec{k}) = \frac{a_3}{\|\vec{a}\|} = \cos\gamma$

$\Rightarrow \quad \hat{\vec{a}}\,(\cos\alpha, \cos\beta, \cos\gamma) \quad \Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Se $a_1 > 0 \Rightarrow \sphericalangle(\vec{a}, \vec{i}) < \frac{\pi}{2}$ (acuto)
$a_1 = 0 \Rightarrow \sphericalangle(\vec{a}, \vec{i}) = \pi/2$ (retto)
$a_1 < 0 \Rightarrow \sphericalangle(\vec{a}, \vec{i}) > \pi/2$ (ottuso)

Se $a_2 > 0 \Rightarrow \sphericalangle(\vec{a}, \vec{j}) < \frac{\pi}{2}$ (acuto)
$a_2 = 0 \Rightarrow \sphericalangle(\vec{a}, \vec{j}) = \pi/2$ (retto)
$a_2 < 0 \Rightarrow \sphericalangle(\vec{a}, \vec{j}) > \pi/2$ (ottuso)

Se $a_3 > 0 \Rightarrow \sphericalangle(\vec{a}, \vec{k}) < \frac{\pi}{2}$ (acuto)
$a_3 = 0 \Rightarrow \sphericalangle(\vec{a}, \vec{k}) = \pi/2$ (retto)
$a_3 < 0 \Rightarrow \sphericalangle(\vec{a}, \vec{k}) > \frac{\pi}{2}$ (ottuso)

Esempio $\vec{a}(2, 0, -7) \Rightarrow \sphericalangle(\vec{a}, \vec{i})$ è acuto
$\sphericalangle(\vec{a}, \vec{j})$ è retto
$\sphericalangle(\vec{a}, \vec{k})$ è ottuso

$\boxed{1}$ **Dati:** $\vec{a}, \vec{b}, \vec{c}$

Trovate: a) $\vec{a}\cdot\vec{b}$; $\vec{b}\cdot\vec{c}$; $\vec{c}\cdot\vec{a}$

b) $\|\vec{a}\|$, $\|\vec{b}\|$, $\|\vec{c}\|$

c) $\hat{\vec{a}}$, $\hat{\vec{b}}$, $\hat{\vec{c}}$

d) $\measuredangle(\vec{a},\vec{b})$; $\measuredangle(\vec{b},\vec{c})$; $\measuredangle(\vec{c},\vec{a})$

e) $$\left.\begin{array}{l} \measuredangle(\vec{a},\vec{i});\ \measuredangle(\vec{a},\vec{j});\ \measuredangle(\vec{a},\vec{k}) \\ \measuredangle(\vec{b},\vec{i});\ \measuredangle(\vec{b},\vec{j});\ \measuredangle(\vec{b},\vec{k}) \\ \measuredangle(\vec{c},\vec{i});\ \measuredangle(\vec{c},\vec{j});\ \measuredangle(\vec{c},\vec{k}) \end{array}\right\}$$ è acuto, retto o ottuso ?!

f) $\vec{np}_{\vec{b}}\,\vec{a}$; $\vec{np}_{\vec{a}}\,\vec{b}$; $\vec{np}_{\vec{a}}\,\vec{c}$; $\vec{np}_{\vec{b}}\,\vec{c}$

(A) $\vec{a}(1,-2,2)$; $\vec{b}(0,4,-3)$; $\vec{c}(3,0,0)$

f) $\vec{np}_{\vec{b}}\,\vec{a} = ?$

$$\|\vec{b}\| = \sqrt{16+9} = \sqrt{25} = 5 \Rightarrow \hat{\vec{b}} = \left(0, \frac{4}{5}, \frac{-3}{5}\right)$$

$$m = \vec{a}\cdot\hat{\vec{b}} = 0 - \frac{8}{5} - \frac{6}{5} = -\frac{14}{5}$$

$$\vec{np}_{\vec{b}}\,\vec{a} = m\cdot\hat{\vec{b}} = \left(0, -\frac{56}{25}, \frac{42}{25}\right)$$

(B) $\vec{a}(-1,0,3)$ $\vec{b}(1,2,-4)$ $\vec{c}(0,1,-1)$

(C) $\vec{a}(2,-1,3)$ $\vec{b}(-2,1,-3)$ $\vec{c}(4,-2,6)$

(D) $\vec{a}(3,0,-4)$ $\vec{b}(2,-1,-2)$ $\vec{c}(-1,4,-2)$

(E) $\vec{a}(-6,0,8)$ $\vec{b}(-2,1,2)$ $\vec{c}(-8,1,10)$

2 ~~Dato~~ Dato: ΔABC

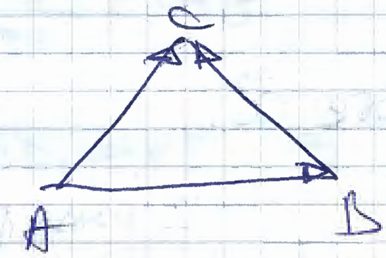
Trovate

a) il tipo del triangolo rispetto ai lati (equilatero; isoscele; scaleno)

b) il tipo del triangolo rispetto agli angoli (acutangolo; rettangolo; ottusangolo)

c) Il perimetro del triangolo

(A) $A(-1,3,0)$; $B(0,1,-2)$; $C(2,3,4)$



$\overrightarrow{AB}(1,-2,-2)$ $\qquad |\overrightarrow{AB}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$\overrightarrow{AC}(3,0,4)$ $\qquad |\overrightarrow{AC}| = \sqrt{9+0+16} = \sqrt{25} = 5$

$\overrightarrow{BC}(2,2,6)$ $\qquad |\overrightarrow{BC}| = \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11}$

$\Rightarrow \Delta ABC$ è scaleno

$P_{\Delta ABC} = 3+5+2\sqrt{11} = 8+2\sqrt{11} = 2(4+\sqrt{11})$

$$\cos\measuredangle A = \cos\measuredangle(\overrightarrow{AB},\overrightarrow{AC}) = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\cdot|\overrightarrow{AC}|} = \frac{-5}{3\cdot 5} = -\frac{1}{3} < 0$$

$\overrightarrow{AB}\cdot\overrightarrow{AC} = 3+0-8 = -5$

$\Rightarrow \measuredangle A$ è ottuso $\Rightarrow \Delta ABC$ è ottusangolo

!!! Se $\cos\measuredangle A > 0 \Rightarrow \measuredangle A$ è acuto e non possiamo dire il tipo del triangolo. Dobbiamo trovare $\cos\measuredangle B = \cos\measuredangle(\overrightarrow{BA},\overrightarrow{BC})$ Se $\cos\measuredangle B > 0$ dobbiamo trovare e $\cos\measuredangle C = \cos\measuredangle(\overrightarrow{CA},\overrightarrow{CB})$

!!!

(B) $A(1;0;-1)$; $B(1;-2;3)$ $C(\sqrt{10}+1;1;2)$

(C) $A(-1,-2,-3)$; $B(-1,2,0)$; $C(0,6,2\sqrt{2})$

(D) $A(-1,-2,-3)$; $B(-1,2,0)$ $C(0,-2,2\sqrt{2})$

(E) $A(-1,2,4)$; $B(0,-1,3)$; $C(-3,8,6)$

!!! Non esiste ΔABC!!! Perché?

3 Trovare vettore $\vec{x}$.

(1) $\vec{x} \begin{cases} \subset xOz \\ \perp \vec{a}(-1,7,2) \\ |\vec{x}|=3 \\ \measuredangle(\vec{x},\vec{u}) \text{ è ottuso} \end{cases}$ $\subset$ passa per ($\vec{x}$ è complanario con il piano xOz)

1) $\vec{x} \subset xOz \Rightarrow \vec{x}(x,0,z)$

2) $\vec{x} \perp \vec{a} \Rightarrow \vec{x}\cdot\vec{a}=0 \Rightarrow -x+0+2z=0$ $\boxed{x=2z}$

$\Rightarrow \vec{x}(2z,0,z)$

3) $|\vec{x}|=\sqrt{4z^2+z^2}=\sqrt{5}|z|$

ma $|\vec{x}|=3 \Rightarrow \sqrt{5}|z|=3$ $|z|=\frac{3}{\sqrt{5}}$

$z_{1,2}=\pm\frac{3}{\sqrt{5}}$

4) $\measuredangle(\vec{x},\vec{u})$ è ottuso $\Rightarrow z<0 \Rightarrow z=-\frac{3}{\sqrt{5}}$

$\Rightarrow \vec{x}\left(\frac{-6}{\sqrt{5}},0,\frac{-3}{\sqrt{5}}\right)$

② $\vec{x}\left|\begin{array}{l} \in xOy \\ \perp \vec{a}(-2,3,1) \\ |\vec{x}| = 7 \\ \measuredangle(\vec{x},\vec{j}) \text{ è acuto} \end{array}\right.$

③ $\vec{x}\left|\begin{array}{l} \parallel Oy \\ \measuredangle(\vec{x},\vec{j}) \text{ è ottuso} \\ |\vec{x}| = 7 \end{array}\right.$

IV Prodotto vettoriale - vettore

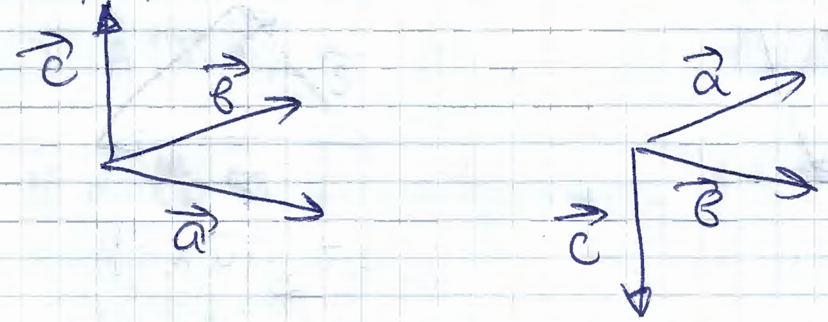
esiste solo nello spazio!!!

① $\vec{c} = \vec{a} \wedge \vec{b}$ o $\vec{c} = \vec{a} \times \vec{b}$

1) $\vec{c}\left|\begin{array}{l} \perp \vec{a} \\ \perp \vec{b} \end{array}\right.$ 2) $\|\vec{c}\| = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \operatorname{sen} \measuredangle(\vec{a},\vec{b})$

$0 \le \measuredangle(\vec{a},\vec{b}) \le \pi$

3) $\vec{a}, \vec{b}, \vec{c}$ destroso



② $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

③ $\vec{a}(a_1, a_2, a_3)$ $\vec{b}(b_1, b_2, b_3)$

$$\vec{a} \times \vec{b} = \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\vec{a}(2,-3,-1)$

$\vec{b}(-1,4,-5)$

$\vec{a}\times\vec{b} = \left(\begin{vmatrix} -3 & -1 \\ 4 & -5 \end{vmatrix}, -\begin{vmatrix} 2 & -1 \\ -1 & -5 \end{vmatrix}, \begin{vmatrix} 2 & -3 \\ -1 & 4 \end{vmatrix} \right)$ I modo

$\vec{a}\times\vec{b} = (19, 11, 5)$

II modo $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & -1 \\ -1 & 4 & -5 \end{vmatrix} =$

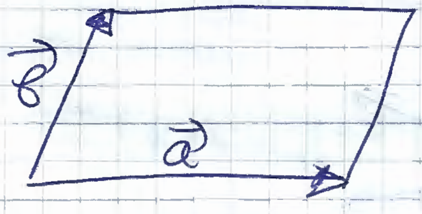
$= \vec{i}\cdot A_{11} + \vec{j}\cdot A_{12} + \vec{k}\cdot A_{13} =$

$= \vec{i}\cdot(-1)^2\cdot\Delta_{11} + \vec{j}\cdot(-1)^2\cdot\Delta_{12} + \vec{k}\cdot(-1)^4\cdot\Delta_{13} =$

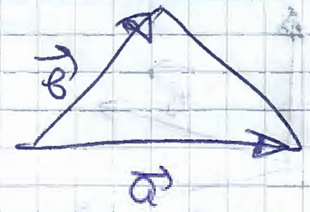
$= \vec{i}\cdot \begin{vmatrix} -3 & -1 \\ 4 & -5 \end{vmatrix} + (-1)\cdot\vec{j}\cdot \begin{vmatrix} 2 & -1 \\ -1 & -5 \end{vmatrix} + \vec{k}\cdot \begin{vmatrix} 2 & -3 \\ -1 & 4 \end{vmatrix} =$

$= 19\vec{i} + 11\vec{j} + 5\vec{k} \Rightarrow \vec{a}\times\vec{b}(19, 11, 5)$

(4)



$S = |\vec{a}\times\vec{b}|$



$S = \frac{1}{2}|\vec{a}\times\vec{b}|$

(5) Se $\vec{a} \parallel \vec{b} \Rightarrow \vec{a}\times\vec{b} = \vec{0}$

Se $\vec{a}\times\vec{b} = \vec{0} \Rightarrow \vec{a} \parallel \vec{b}$ $(\vec{a}\neq\vec{0};\ \vec{b}\neq\vec{0})$
o $\vec{a}=\vec{0}$ o $\vec{b}=\vec{0}$ o $\vec{a}=\vec{0}$ e $\vec{b}=\vec{0}$

1. È data la retta AB. Dite il punto C è sulla retta o no?

(A) $A(2,-1,3)$ $B(-1,4,-1)$ $C(1,-1,7)$

$\overrightarrow{AB}(-3,5,-4)$

$\overrightarrow{AC}(-1,0,4)$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\left(\begin{vmatrix}5&-4\\0&4\end{vmatrix},-\begin{vmatrix}-3&-4\\-1&4\end{vmatrix},\begin{vmatrix}-3&5\\-1&0\end{vmatrix}\right)=\left(20,-\left|\quad\right|,\left|\quad\right|\right)$$

$\neq\vec{0}\Rightarrow C\notin AB$ (C non è sulla retta)

AB non passa per C

(B) $A(-1,2,-1)$ $B(0,1,-1)$ $C(1,0,-1)$

$\overrightarrow{AB}(1,-1,0)$

$\overrightarrow{AC}(2,-2,0)$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\left(\begin{vmatrix}-1&0\\-2&0\end{vmatrix},-\begin{vmatrix}1&0\\2&0\end{vmatrix},\begin{vmatrix}1&-1\\2&-2\end{vmatrix}\right)=(0,0,0)$$

$\Rightarrow$ C è sulla retta

(C) $A(1,-1,1)$ $B(2,3,-7)$ $C(0,1,4)$

(D) $A(5,1,-1)$ $B(0,1,0)$ $C(10,1,-2)$

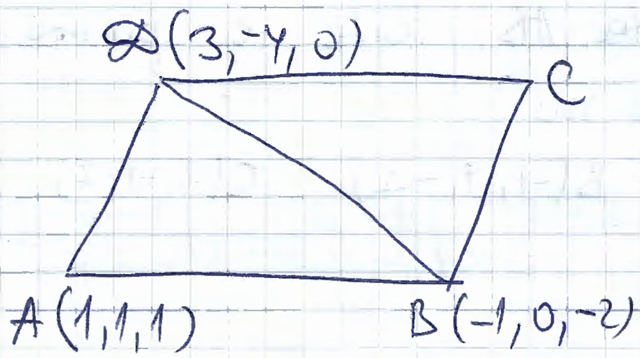
2. È dato il parallelogramma ABCD dove

$A(1,1,1)$; $B(-1,0,-2)$; $D(3,-4,0)$

Trovate: a) S_{ABCD}

b) $S_{\triangle BDC}$

c) le coordinate ~~di p~~ del punto C



a) $\vec{AB}(-2,-1,-3)$

$\vec{AD}(2,-5,-1)$

$$\vec{AB}\times\vec{AD}=\left(\begin{vmatrix}-1 & -3\\ -5 & -1\end{vmatrix}, -\begin{vmatrix}-2 & -3\\ 2 & -1\end{vmatrix}, \begin{vmatrix}-2 & -1\\ 2 & -5\end{vmatrix}\right)=$$

$$=(-14, -8, 12)$$

$$|\vec{AB}\times\vec{AD}|=\sqrt{196+64+144}=\sqrt{404}=2\sqrt{101}$$

$$\Rightarrow S_{ABCD}=2\sqrt{101}$$

b) $S_{\triangle BDC}=S_{\triangle ABD}=\frac{1}{2}S_{ABCD}=\frac{1}{2}\cdot 2\cdot\sqrt{101}=\sqrt{101}$

c) Sia $C(x,y,z)$

$\vec{AB}=\vec{DC}$

$\vec{DC}(x-3, y+4, z)$

$$\Rightarrow \begin{cases} x-3=-2 \\ y+4=-1 \\ z=-3 \end{cases} \quad \begin{matrix} x=1 \\ y=-5 \\ z=-3 \end{matrix} \quad \Rightarrow C(1,-5,-3)$$