(8) $\int_\gamma x\,ds$ γ è la parabola $y = x^2$

tra le $O(0,0)$ e $A(\sqrt{2}, 2)$

(9) $\int_\gamma \frac{x}{y}\,ds$ γ è la parabola $y^2 = 2x$

tra le $A(1, \sqrt{2})$ e $B(2,2)$

Risp. $\frac{1}{6}(5\sqrt{5} - 3\sqrt{3})$

(10) $\int_\gamma y\,ds$ γ è ~~la frontiera del dominio nei~~ della parabola $y^2 = 2px$ tra le punti di intersezione con la parabola $x^2 = 2py$

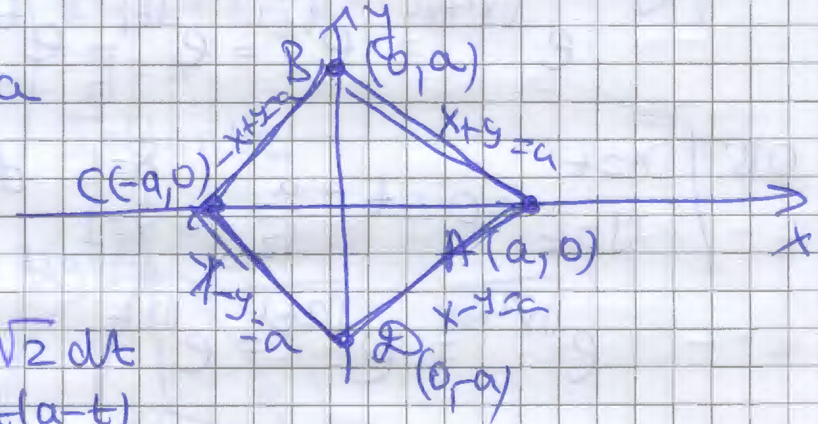
$\mathcal{D}: \begin{cases} y^2 \le 2px \\ x^2 \le 2py \end{cases}$ $p \neq 0$

$p \neq 0$

!!! $p>0$ $p<0$

Risp. $\frac{p^2}{3}(5\sqrt{5} - 1)$ se $p>0$ $\frac{p^2}{3}(1 - 5\sqrt{5})$ se $p<0$

(11) $\int_\gamma xy\,ds$ γ: $|x| + |y| = a$ $a>0$ nel R_2



(I) $x \ge 0$, $y \ge 0$ $x + y = a$

$\begin{cases} x = t \\ y = a - t \end{cases}$ $\dot{x} = 1$, $\dot{y} = -1$

$0 \le t \le a$ $ds = \sqrt{2}\,dt$

$xy = t(a-t)$

(II) $x \le 0$, $y \ge 0$ $-x + y = a$ $\begin{cases} x = t \\ y = a + t \end{cases}$ $-a \le t \le 0$

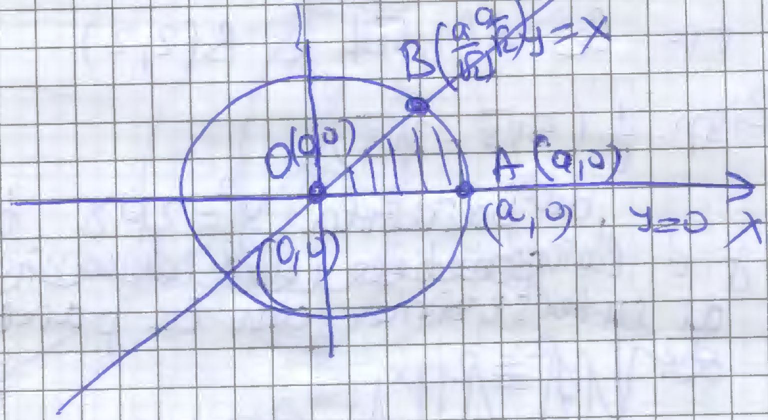
(III) $x \le 0$, $y \le 0$ $-x - y = a$ $\begin{cases} x = t \\ y = -t - a \end{cases}$ $-a \le t \le 0$

(IV) $x \ge 0$, $y \le 0$ $x - y = a$ $\begin{cases} x = t \\ y = t - a \end{cases}$ $0 \le t \le a$

$\boxed{\text{Risp. } 0}$

(12) $\int_\gamma e^{\sqrt{x^2+y^2}}\, ds$ γ è la frontiera del dominio = 22 =

$D: \begin{cases} x^2+y^2 \leq a^2 \\ x \geq y \\ y \geq 0 \\ a > 0 \end{cases}$ nel R_2



$\begin{cases} y = x \\ x^2+y^2 = a^2 \end{cases}$ $2x^2 = a^2$ $x = \frac{a}{\sqrt2}$ $B\left(\frac{a}{\sqrt2}, \frac{a}{\sqrt2}\right)$

OA: $\begin{cases} x = t \\ y = 0 \end{cases}$ $0 \leq t \leq a$ $\dot{x} = 1$; $\dot{y} = 0$ $ds = dt$

$e^{\sqrt{x^2+y^2}} = e^{\sqrt{t^2}} = e^{|t|} = e^t$

OB: $\begin{cases} x = t \\ y = t \end{cases}$ $0 \leq t \leq \frac{a}{\sqrt2}$ $\begin{matrix}\dot{x} = 1 \\ \dot{y} = 1\end{matrix}$ $ds = \sqrt2\, dt$

$e^{\sqrt{x^2+y^2}} = e^{\sqrt{2t^2}} = e^{\sqrt2 t}$

AB: $\begin{cases} x = a\cos t \\ y = a\sin t \end{cases}$

$\begin{cases} a\cos t = a \\ a\sin t = 0 \end{cases}$ $\begin{matrix}\cos t = 1 \\ \sin t = 0\end{matrix} \Rightarrow t = 0$ $\begin{cases} a\cos t = \frac{a}{\sqrt2} \\ a\sin t = \frac{a}{\sqrt2} \end{cases}$ $\begin{matrix}\cos t = \frac{\sqrt2}{2} \\ \sin t = \frac{\sqrt2}{2} \\ t = \pi/4\end{matrix}$ $\boxed{0 \leq t \leq \frac{\pi}{4}}$

$\dot{x} = -a\sin t \qquad \dot{y} = a\cos t$

$$ds = \sqrt{a^2(\sin^2 t + \cos^2 t)}\,dt = a\,dt$$

$$e^{\sqrt{x^2+y^2}} = e^{\sqrt{a^2(\cos^2 t+\sin^2 t)}} = e^a$$

$$\int_\gamma e^{\sqrt{x^2+y^2}}\,ds = \int_0^a e^t\,dt + \int_0^{\frac{a}{\sqrt2}} e^{\sqrt2 t}\cdot\sqrt2\,dt +$$

$$+\int_0^{\frac{\pi}{4}} e^a\cdot a\,dt = e^t\Big|_0^a + e^{\sqrt2 t}\Big|_0^{\frac{a}{\sqrt2}} + a\cdot e^a\cdot t\Big|_0^{\pi/4} =$$

$$= e^a - e^0 + e^a - e^0 + a\cdot e^a\frac{\pi}{4} - 0 =$$

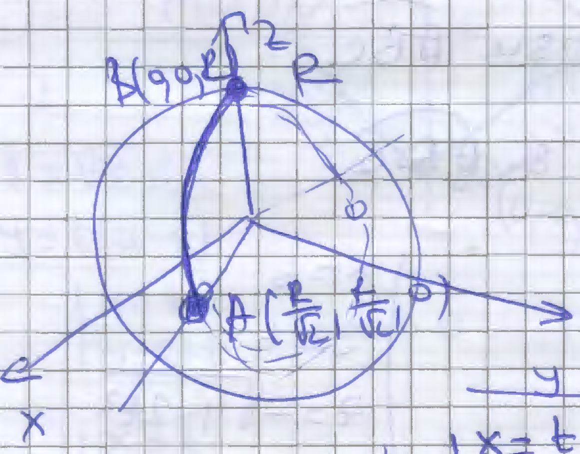
$$= 2e^a - 2 + ae^a\frac{\pi}{4} = \frac{e^a}{4}(8 - a\pi) - 2$$

(13) $\int_\gamma (x+y)\,ds \qquad \gamma: \begin{cases} x^2+y^2+z^2=R^2 \\ y=x \\ \text{I ottante} \end{cases}$

$\begin{cases} y=x \\ z=0 \\ 2x^2=R^2 \end{cases}$

$x = \pm\frac{R}{\sqrt2} \quad x>0$

$x=\frac{R}{\sqrt2} \quad y=\frac{R}{\sqrt2} \quad z=0$



$x=0 \Rightarrow y=0$

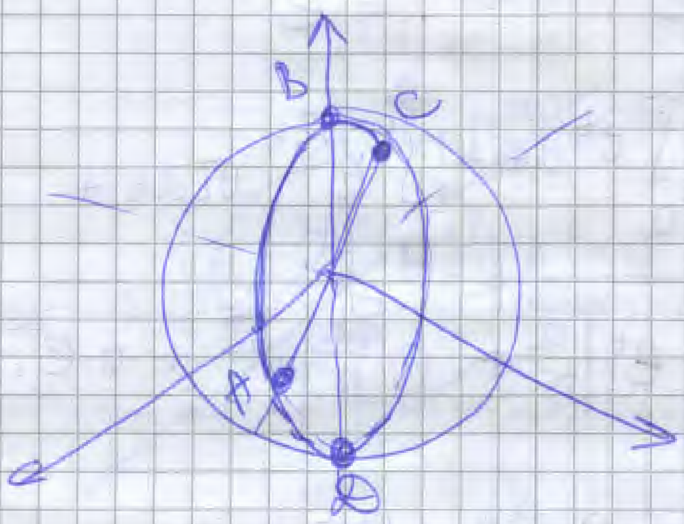
$z^2=R^2 \quad z=\pm R \quad z>0 \quad z=R$

$z^2=R^2-2t^2 \quad z\geq 0$

$z=\sqrt{R^2-2t^2}$

$$\boxed{\begin{cases} x=t \\ y=t \\ z=\sqrt{R^2-2t^2} \end{cases} \quad 0\leq t\leq \frac{R}{\sqrt2}}$$

(17) $\int_\gamma \sqrt{2y^2+z^2}\,ds$ $\quad \gamma: \begin{cases} x^2+y^2+z^2=4 \\ y=x \end{cases}$ $\quad =24=$



$x=0 \Rightarrow y=0$

$z=\pm 2 \Rightarrow B(0,0,2) \quad D(0,0,-2)$

$z=0 \quad x=y \quad 2x^2=4 \quad x^2=2 \quad x=\pm\sqrt{2}$

$A(\sqrt{2},\sqrt{2},0) \quad C(-\sqrt{2},-\sqrt{2},0)$

$\begin{cases} x=t \\ y=t \end{cases} \Rightarrow 2t^2+z^2=4 \quad z^2=4-2t^2$

$z=\sqrt{4-2t^2}$ su $\widehat{ABC}$

$z=-\sqrt{4-2t^2}$ su $\widehat{ADC}$

$\begin{cases} x=t \\ y=t \\ z=\sqrt{4-2t^2} \end{cases}$ $\qquad$ $\begin{cases} x=t \\ y=t \\ z=-\sqrt{4-2t^2} \end{cases}$

$-\sqrt{2}\le t\le\sqrt{2}$ $\qquad$ $-\sqrt{2}\le t\le\sqrt{2}$

$\sqrt{2y^2+z^2}=\sqrt{2t^2+4-2t^2}=2$ $\qquad$ $\sqrt{2y^2+z^2}=\sqrt{2t^2+4-2t^2}=2$

$= 2\sqrt{=}$

$ds = \sqrt{1+1+\left(\frac{-2t}{\sqrt{4-2t^2}}\right)^2}\,dt =$ ~~$ds = \sqrt{1+1+\left(\frac{2t}{\sqrt{4-2t^2}}\right)^2}\,dt$~~

$= \sqrt{2 + \frac{4t^2}{4-2t^2}}\ dt =$

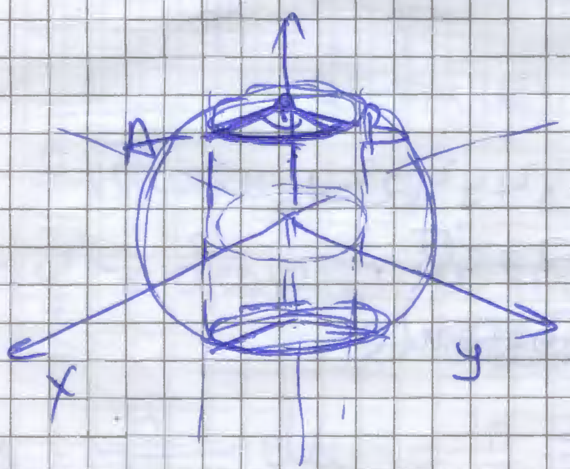
$= \sqrt{\frac{8-4t^2+4t^2}{2(2-t^2)}}\,dt = \sqrt{\frac{4}{2-t^2}}\,dt = \frac{2}{\sqrt{2-t^2}}\,dt$

$\Rightarrow \int_\gamma \sqrt{2y^2+z^2}\,ds = 2\int_{-\sqrt{2}}^{\sqrt{2}} 2\cdot\frac{2}{\sqrt{2-t^2}}\,dt =$

$= \frac{8}{\sqrt{2}}\cdot\sqrt{2}\int_{-\sqrt{2}}^{\sqrt{2}} \frac{1}{\sqrt{1-\left(\frac{t}{\sqrt{2}}\right)^2}}\,d\left(\frac{t}{\sqrt{2}}\right) = 8\arcsin\frac{t}{\sqrt{2}}\Big|_{-\sqrt{2}}^{\sqrt{2}} =$

$= 8\left(\arcsin 1 - \arcsin(-1)\right) = 16\arcsin 1 = 16\cdot\frac{\pi}{2} = \underline{\underline{8\pi}}$ ||

(15) $\int_\gamma xyz\,ds \qquad \gamma: \begin{cases} x^2+y^2+z^2 = r^2 \\ x^2+y^2 = \frac{1}{4}r^2 \\ \text{I ottante} \end{cases}$



$x^2+y^2 = \frac{1}{4}r^2$

$\Rightarrow \frac{1}{4}r^2 + z^2 = r^2$

$z^2 = \frac{3}{4}r^2$

$z = \pm\frac{\sqrt{3}}{2}r$

$z \geq 0 \quad \boxed{z = \frac{\sqrt{3}}{2}r} \Rightarrow$

$$\begin{cases} x = \frac{1}{2} r \cos t \\ y = \frac{1}{2} r \sin t \\ z = \frac{\sqrt{3}}{2} r \end{cases}$$

$$0 \le t \le \frac{\pi}{2}$$

$\dot{x} = -\frac{1}{2} r \sin t \quad \dot{y} = \frac{1}{2} r \cos t \quad \dot{z} = 0$

$$ds = \sqrt{\frac{1}{4} r^2 (\sin^2 t + \cos^2 t)}\, dt = \frac{1}{2} r\, dt$$

$$xyz = \frac{\sqrt{3}}{8} r^3 \sin t \cos t$$

$$\int_\gamma xyz\, ds = \int_0^{\pi/2} \frac{\sqrt{3}}{8} r^3 \cdot \frac{1}{2} r \cdot \sin t \cos t\, dt =$$

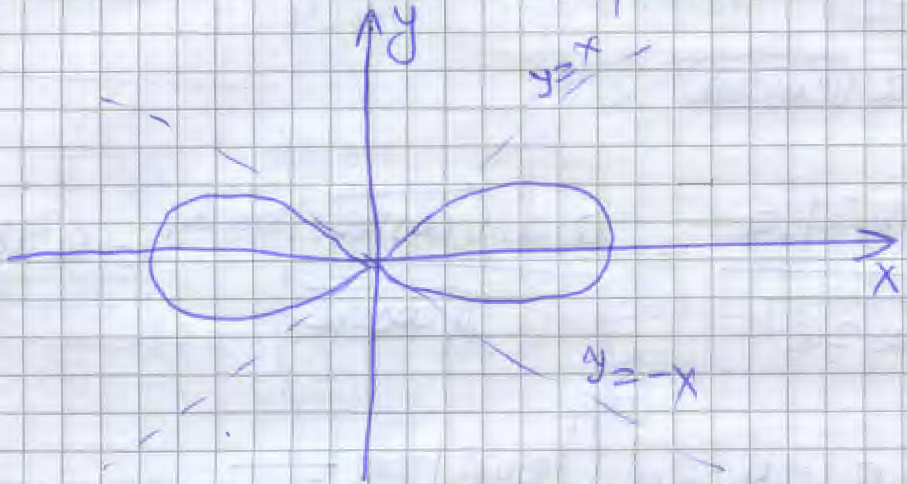
$$= \frac{\sqrt{3}}{16} r^4 \int_0^{\pi/2} \sin t\, d(\sin t) = \frac{\sqrt{3}}{16} \cdot r^4 \cdot \frac{\sin^2 t}{2} \Big|_0^{\pi/2} =$$

$$= \frac{\sqrt{3}\, r^4}{32}$$

(16) $\int xyz\, ds$ $\quad \gamma: \begin{cases} x^2 + y^2 + z^2 = 16 \\ z = 2\sqrt{3} \\ \text{Istante} \end{cases}$

(17) $\int_\gamma |y|\,ds$ $\quad \gamma: (x^2+y^2)^2 = a^2(x^2-y^2)$

nel piano $\quad a>0$



$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases} \qquad \begin{aligned} x^2+y^2 &= \rho^2 \\ x^2-y^2 &= \rho^2(\cos^2\theta - \sin^2\theta) = \rho^2\cos 2\theta \end{aligned}$$

$$\rho^4 = a^2\rho^2\cos 2\theta \qquad \boxed{\rho^2 = a^2\cos 2\theta} \quad \text{ma } \rho \geq 0$$

l'equazione polare $\quad$!!! solo nel piano !!!

$$\Rightarrow \rho = \sqrt{a^2\cos 2\theta} \quad \cup \quad \cancel{\rho = \sqrt{-a\cos 2\theta}}$$

$$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \qquad \cancel{\frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}}$$

$$\cup \qquad \cancel{\rho^2 = a\cos 2\theta}$$

$$\frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}$$

$$\rho' = \frac{1}{2\sqrt{a^2\cos 2\theta}} \cdot a^2(-\sin 2\theta)\cdot 2 = \frac{-a^2\sin 2\theta}{\sqrt{a^2\cos 2\theta}}$$

$$\rho^2 + \rho'^2 = a^2\cos 2\theta + \frac{a^4\sin^2 2\theta}{a^2\cos 2\theta} = \frac{a^4}{a^2\cos 2\theta} = \frac{a^2}{\cos 2\theta}$$

$$ds = \frac{a}{\sqrt{\cos 2\theta}}\,d\theta \qquad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \quad \cup \quad \frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}$$

$$\int_\gamma |y|\,ds = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{|\rho \sin\theta| \cdot a}{\sqrt{\cos 2\theta}}\,d\theta + \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \frac{|\rho \sin\theta| \cdot a}{\sqrt{\cos 2\theta}}\,d\theta =$$

$$\rho = a\sqrt{\cos 2\theta}$$

$$\frac{|\rho \sin\theta| \cdot a}{\sqrt{\cos 2\theta}} = \frac{a \cdot a \cdot \sqrt{\cos 2\theta} \cdot |\sin\theta|}{\sqrt{\cos 2\theta}} = a^2 |\sin\theta|$$

$$= a^2 \int_{-\pi/4}^{\pi/4} |\sin\theta|\,d\theta + a^2 \int_{3\pi/4}^{5\pi/4} |\sin\theta|\,d\theta =$$

$-\frac{\pi}{4} \le \theta \le 0 \quad |\sin\theta| = -\sin\theta$

$0 \le \theta \le \frac{\pi}{4} \quad |\sin\theta| = \sin\theta$

$\frac{3\pi}{4} \le \theta \le \pi \quad |\sin\theta| = \sin\theta$

$\pi \le \theta \le \frac{5\pi}{4} \quad |\sin\theta| = -\sin\theta$

$$= a^2 \left[\int_0^{\frac{\pi}{4}} \sin\theta\,d\theta + \int_{\frac{3\pi}{4}}^{\pi} \sin\theta\,d\theta - \int_{\pi/4}^{0} \sin\theta\,d\theta - \int_{\pi}^{5\pi/4} \sin\theta\,d\theta \right] =$$

$$= a^2 \left[-\cos\theta \Big|_0^{\frac{\pi}{4}} - \cos\theta \Big|_{\frac{3\pi}{4}}^{\pi} + \cos\theta \Big|_{\frac{\pi}{4}}^{0} + \cos\theta \Big|_{\pi}^{\frac{5\pi}{4}} \right] =$$

$$= a^2 \left(-\cancel{\frac{\sqrt{2}}{2}} + 1 - (-1) + \cancel{\frac{\sqrt{2}}{2}} + 1 - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - (-1) \right) =$$

$$= a^2 (1 + 1 + 1 + 1 - 2\sqrt{2}) = a^2 (4 - 2\sqrt{2}) = 2a^2 (2 - \sqrt{2})$$

18 Calcolare $\int_\gamma (x^2+y^2)^2\,ds$ essendo γ la curva di equazione polare

$$\rho = e^{2\theta} \qquad \theta \in (-\infty, 0]$$

(19) II modo (2) dalla pag. 13

$$\int_\gamma \frac{ds}{x-y}$$

γ: $y=\frac{x}{2}-2$ $\quad A(0,-2)$; $B(4,0)$

$$y'=\frac{1}{2} \qquad ds=\sqrt{1+\frac{1}{4}}\,dx=\frac{\sqrt{5}}{2}dx$$

$$0\le x\le 4$$

$$x-y= x-\frac{x}{2}+2=\frac{x}{2}+2=\frac{x+4}{2}$$

$$\Rightarrow \int_\gamma \frac{ds}{x-y}=\int_0^4 \frac{\frac{\sqrt{5}}{2}}{\frac{x+4}{2}}dx=\sqrt{5}\int_0^4\frac{1}{x+4}d(x+4)=$$

$$=\sqrt{5}\ln|x+4|\Big|_0^4=\sqrt{5}\ln 8-\sqrt{5}\ln 4=\sqrt{5}\ln\frac{8}{4}=$$

$$=\sqrt{5}\ln 2$$

solo nel piano!!!

Applicazioni

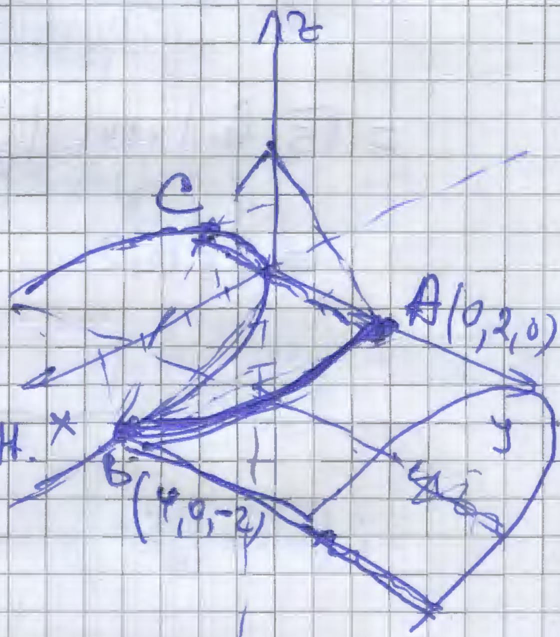
(I) Lunghezza della curva

È data la curva γ [$\gamma: \vec{r}(t)$; $t\in[a,b]$ regolare]

$$\boxed{\text{La lunghezza } L_\gamma = \int_\gamma ds}$$

(1) Calcolate la lunghezza della curva γ

$$\gamma: \begin{cases} z^2 = x \\ x+y+z=2 \\ \underline{V} \text{ ottante} \end{cases}$$



Punti di intersezione

$\underline{V}$ ottante $\quad x\geq 0 \; y\geq 0 \; z\leq 0$

$x=0 \Rightarrow z=0 \Rightarrow y=2 \quad A(0,2,0) \in \underline{V}$ ott.

$y=0 \quad x+0+z=2$

$$\begin{cases} x = 2-z \\ z^2 = x \end{cases}$$

$z^2 = 2-z$

$z^2+z-2=0$

$z_{1,2} = \dfrac{-1\pm 3}{2}$ $\nearrow -2 \Rightarrow B(4,0,-2) \in \underline{V}$ ott. $\searrow 1 \Rightarrow C(1,0,1) \notin$ V ott.

La nostra curva è AB

$\boxed{z=t} \Rightarrow \boxed{x=t^2} \Rightarrow t^2+t+y=2 \Rightarrow y=2-t-t^2 = 31 =$

$$\gamma: \left| \begin{array}{l} x=t^2 \\ y=2-t-t^2 \\ z=t \end{array} \right.$$

$-2 \leq t \leq 0$

$$\boxed{L_\gamma = \int_\gamma ds}$$

$\dot{x}=2t \quad \dot{y}=-1-2t \quad \dot{z}=1$

$$ds = \sqrt{4t^2+1+4t+4t^2+1}\,dt = \sqrt{8t^2+4t+2}\,dt$$

$$L_\gamma = \int_{-2}^{0} \sqrt{8t^2+4t+2}\,dt$$

$$t = x - \frac{4}{2.8} = x - \frac{1}{4} \qquad \boxed{dx = dt}$$

$x = t+\frac{1}{4} \qquad t=-2 \quad x = -2+\frac{1}{4} = -\frac{7}{4}$

$t=0 \quad x = 0+\frac{1}{4} = \frac{1}{4}$

$$\sqrt{8t^2+4t+2} = \sqrt{8\left(x-\frac{1}{4}\right)^2+4\left(x-\frac{1}{4}\right)+2} =$$

$$= \sqrt{8x^2-4x+\frac{1}{2}+4x-1+2} = \sqrt{8x^2+\frac{3}{2}}$$

$$\Rightarrow L_\gamma = \int_{-\frac{7}{4}}^{1/4} \sqrt{8x^2+\frac{3}{2}}\,dx = x \cdot \sqrt{8x^2+\frac{3}{2}}\,\Bigg|_{-\frac{7}{4}}^{\frac{1}{4}} - \int_{-\frac{7}{4}}^{1/4} x \cdot \frac{1 \cdot 16x}{2\sqrt{8x^2+\frac{3}{2}}}\,dx =$$

$$L_\gamma = \frac{1}{4}\cdot\sqrt{8\cdot\frac{1}{16}+\frac{3}{2}} - \left(-\frac{7}{4}\right)\cdot\sqrt{8\cdot\frac{49}{16_2}+\frac{3}{2}} - \quad = 32 =$$

$$- \int_{-\frac{7}{4}}^{\frac{1}{4}} \frac{8x^2}{\sqrt{8x^2+\frac{3}{2}}}dx = \frac{1}{4}\cdot\sqrt{2} + \frac{7}{4}\sqrt{26} - \int_{-7/4}^{\frac{1}{4}} \frac{8x^2+\frac{3}{2}-\frac{3}{2}}{\sqrt{8x^2+\frac{3}{2}}}dx =$$

$$= \frac{\sqrt{2}+7\sqrt{26}}{4} - \int_{-7/4}^{1/4}\sqrt{8x^2+\frac{3}{2}}\,dx + \frac{3}{2}\int_{-7/4}^{1/4}\frac{dx}{\sqrt{8x^2+\frac{3}{2}}} =$$

$$= \frac{\sqrt{2}+7\sqrt{26}}{4} - L_\gamma + \frac{3}{2\cdot 2\sqrt{2}}\int_{-7/4}^{1/4}\frac{1}{\sqrt{(2\sqrt{2}x)^2+\frac{3}{2}}}d(2\sqrt{2}x)$$

$$2L_\gamma = \frac{\sqrt{2}+7\sqrt{26}}{4} + \frac{3}{4\sqrt{2}}\ln\left|2\sqrt{2}x+\sqrt{8x^2+\frac{3}{2}}\right|\Bigg|_{-\frac{7}{4}}^{\frac{1}{4}}$$

$$L_\gamma = \frac{\sqrt{2}+7\sqrt{26}}{8} + \frac{3}{8\sqrt{2}}\left[\ln\left(\frac{2\sqrt{2}}{4}+\sqrt{2}\right) - \ln\left|-\frac{2\sqrt{2}\cdot 7}{4}+\sqrt{26}\right|\right] =$$

$$= \frac{\sqrt{2}+7\sqrt{26}}{8} + \frac{3}{8\sqrt{2}}\left[\ln\left(\frac{\sqrt{2}}{2}+\sqrt{2}\right) - \ln\left(-\frac{7\sqrt{2}}{2}+\sqrt{26}\right)\right] =$$

$$= \frac{\sqrt{2}+7\sqrt{26}}{8} + \frac{3}{8\sqrt{2}}\left[\ln\frac{3\sqrt{2}}{2} - \ln\frac{2\sqrt{26}-7\sqrt{2}}{2}\right] =$$

$$= \frac{\sqrt{2}+7\sqrt{26}}{8} + \frac{3}{8\sqrt{2}}\ln\frac{\frac{3\sqrt{2}}{2}}{\frac{2\sqrt{26}-7\sqrt{2}}{2}} =$$

$$= \frac{2+7\sqrt{52}}{8\sqrt{2}} + \frac{3}{8\sqrt{2}}\ln\frac{3\sqrt{2}(2\sqrt{26}+7\sqrt{2})}{4\cdot 26-49\cdot 2} =$$

$$L_\gamma = \frac{2+14\sqrt{13}}{8\sqrt{2}} + \frac{3}{8\sqrt{2}} \ln \frac{6\sqrt{52}+21\cdot 2}{104-98} = \qquad = 33 =$$

$$= \frac{2+14\sqrt{13}}{8\sqrt{2}} + \frac{3}{8\sqrt{2}} \ln \frac{12\sqrt{13}+42}{6} = \frac{2+14\sqrt{13}}{8\sqrt{2}} + \frac{3}{8\sqrt{2}} \cancel{\ldots}$$

$$+ \frac{3}{8\sqrt{2}} \ln \frac{6(2\sqrt{13}+7)}{6} = \frac{1}{8\sqrt{2}}\left(2+14\sqrt{13}+3\ln(2\sqrt{13}+7)\right)$$

II La Massa totale di una linea materiale

Sia γ una linia materiale di densità lineare δ

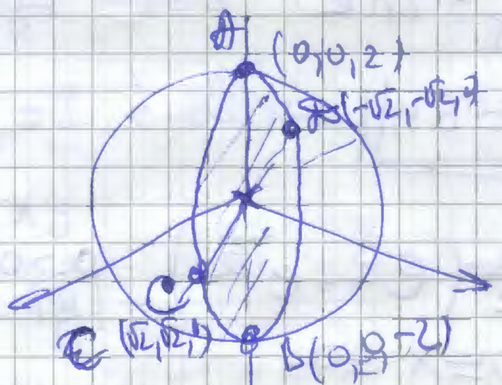
$$\Rightarrow \boxed{m = \int_\gamma \delta \, ds}$$

Se la linia materiale è omogenea $\Rightarrow$ la densità lineare δ e costante

$$\Rightarrow m = \int_\gamma \delta \, ds = \delta \int_\gamma ds = \boxed{\delta \cdot L_\gamma}$$

① $\gamma : \begin{cases} x^2+y^2+z^2=4 \\ y=x \end{cases}$ $\qquad \delta(x,y,z) = \sqrt{2y^2+z^2}$

$m = ?$



$y=x$ è il piano che passa per il centro $O(0,0,0)$ della sfera $\Rightarrow$ γ è una circonferenza con il centro $O(0,0,0)$ e il raggio $R=2$

$$m = \int_\gamma \sqrt{2y^2+z^2}\, ds$$

$x=0 \Rightarrow y=0 \Rightarrow z^2=4 \Rightarrow z=\pm 2 \Rightarrow A(0,0,2)$; $\cancel{\quad}$ $= 34 =$
e p. $B(0,0,-2)$

$\boxed{z=0}$ $y=x$ $2x^2=4$ $x^2=2$ $x=\pm\sqrt{2}$

$\Rightarrow C(\sqrt{2},\sqrt{2},0)$ $D(-\sqrt{2},-\sqrt{2},0)$

$\Rightarrow$ Sia $x=t \Rightarrow y=t$

$\Rightarrow t^2+t^2+z^2=4$ $z^2=4-2t^2$

$z=\pm\sqrt{4-2t^2}$

γ_{CAD} $z\geq 0$ $\begin{cases} x=t \\ y=t \\ z=\sqrt{4-2t^2} \end{cases}$ $-\sqrt{2}\leq t\leq\sqrt{2}$

γ_{CBD} : $z\leq 0$ $\begin{cases} x=t \\ y=t \\ z=-\sqrt{4-2t^2} \end{cases}$ $-\sqrt{2}\leq t\leq\sqrt{2}$

$\delta=\sqrt{2y^2+z^2}=\sqrt{t^2+t^2+4-2t^2}=\sqrt{4}=2 \Rightarrow$
la densità è costante per tutte e due curve.

$\Rightarrow m=\delta\cdot L_\gamma=2\cdot L_\gamma$
ma $L_\gamma=2\pi R$ $R=2$ $L_\gamma=4\pi$

$\Rightarrow \underline{\underline{m=8\pi}}$

(2) $\gamma: y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ $a>0$ nel piano
$0\leq x\leq a$
$\delta(x,y)=\frac{1}{y}$ $m=?$

$$m=\int_\gamma \frac{1}{y}\,ds$$

$$\frac{1}{y}=\frac{1}{\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)}$$

$$y'=\frac{a}{2}\left(e^{\frac{x}{a}}\cdot\frac{1}{a}-e^{-\frac{x}{a}}\cdot\frac{1}{a}\right)=\frac{1}{2}\left(e^{\frac{x}{a}}-e^{-\frac{x}{a}}\right)$$

$$1+y'^2=1+\frac{1}{4}\left(e^{\frac{x}{a}}-e^{-\frac{x}{a}}\right)=\frac{4+e^{\frac{2x}{a}}-2+e^{-\frac{2x}{a}}}{4}=$$

$$=\frac{e^{\frac{2x}{a}}+2\cdot e^{\frac{x}{a}}\cdot e^{-\frac{x}{a}}+e^{-\frac{2x}{a}}}{4}=\left(\frac{e^{\frac{x}{a}}+e^{-\frac{x}{a}}}{2}\right)^2 >0$$

$$ds=\sqrt{\left(\frac{e^{\frac{x}{a}}+e^{-\frac{x}{a}}}{2}\right)^2}\,dx=\frac{e^{\frac{x}{a}}+e^{-\frac{x}{a}}}{2}\,dx$$

$$\Rightarrow m=\int_0^a \frac{1}{\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)}\cdot\frac{e^{\frac{x}{a}}+e^{-\frac{x}{a}}}{2}\,dx=\frac{1}{a}\int_0^a dx=$$

$$=\frac{1}{a}x\Big|_0^a=\frac{a}{a}-0=\underline{\underline{1}}$$

③ $\gamma:\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ é ellisse $m=?$ $a>0$ $b>0$ $a\neq b$

a) $\delta(x,y)=xy$

b) $\delta(x,y)=|y|$

$$\gamma:\begin{cases} x=a\cos t\\ y=a\sin t\\ 0\le t\le 2\pi\end{cases}$$

б) $m = \int\limits_{\gamma} |y|\,ds$

$|y| = |b\sin t| = b\,|\sin t|$

$\dot{x} = -a\sin t \qquad \dot{y} = b\cos t$

$ds = \sqrt{a^2\sin^2 t + b^2\cos^2 t}\,dt$

$$m = \int_0^{2\pi} b\cdot|\sin t|\cdot\sqrt{a^2\sin^2 t + b^2\cos^2 t}\,dt =$$

$$= b\int_0^{\pi} \sin t\cdot\sqrt{a^2\sin^2 t + b^2\cos^2 t}\,dt - b\int_{\pi}^{2\pi} \sin t\cdot\sqrt{a^2\sin^2 t + b^2\cos^2 t}\,dt =$$

$$= -b\int_0^{\pi} \sqrt{a^2(1-\cos^2 t) + b^2\cos^2 t}\,d(\cos t) + b\int_{\pi}^{2\pi} \sqrt{a^2(1-\cos^2 t) + b^2\cos^2 t}\,d(\cos t)$$

Pogušimo $\boxed{\cos t = x}$ $\quad t=0 \quad x=1$

$t=\pi \quad x=-1$

$t=2\pi \quad x=1$

$$= -b\int_1^{-1} \sqrt{a^2+(b^2-a^2)x^2}\,dx + b\int_{-1}^{1} \sqrt{a^2+(b^2-a^2)x^2}\,dx$$

$$I = \int \sqrt{(b^2-a^2)x^2+a^2}\,dx = x\cdot\sqrt{(b^2-a^2)x^2+a^2} - \int x\cdot\frac{1\cdot 2(b^2-a^2)x}{2\sqrt{(b^2-a^2)x^2+a^2}}\,dx =$$

$$= x\cdot\sqrt{(b^2-a^2)x^2+a^2} - \int \frac{(b^2-a^2)x^2+a^2-a^2}{\sqrt{(b^2-a^2)x^2+a^2}}\,dx =$$

$$= x\cdot\sqrt{(b^2-a^2)x^2+a^2} - I + a^2\int \frac{1}{\sqrt{a^2+(b^2-a^2)x^2}}\,dx$$

!!! Se $a<b \Rightarrow a^2<b^2$ $(a>0, b>0)$

$\Rightarrow \boxed{b^2-a^2>0}$

$$\Rightarrow \int \frac{1}{\sqrt{(b^2-a^2)x^2+a^2}}\,dx = \frac{1}{\sqrt{b^2-a^2}}\int \frac{1}{\sqrt{(\sqrt{b^2-a^2}\,x)^2+a^2}}\,d\left[\sqrt{b^2-a^2}\,x\right]$$

$$= \frac{1}{\sqrt{b^2-a^2}}\ln\left|\sqrt{b^2-a^2}\,x+\sqrt{(b^2-a^2)x^2+a^2}\right|$$

Se $a>b \Rightarrow a^2>b^2 \Rightarrow a^2-b^2>0 \Rightarrow b^2-a^2<0$

$$\frac{1}{\sqrt{a^2-b^2}}\int \frac{1}{\sqrt{a^2-(\sqrt{a^2-b^2}\,x)^2}}\,d\left(\sqrt{a^2-b^2}\,x\right) =$$

$$= \boxed{\frac{1}{\sqrt{a^2-b^2}}\arcsen\frac{\sqrt{a^2-b^2}\,x}{a}}$$

$$2I = x\cdot\sqrt{(b^2-a^2)x^2+a^2} + \begin{cases} \dfrac{a^2}{\sqrt{b^2-a^2}}\ln\left|\sqrt{b^2-a^2}\,x+\sqrt{(b^2-a^2)x^2+a^2}\right| & \text{se } a<b \\[2ex] \dfrac{a^2}{\sqrt{a^2-b^2}}\arcsin\dfrac{\sqrt{a^2-b^2}\,x}{a} & \text{se } \underline{a>b} \end{cases}$$

Calcolate la massa per $a=4$ $b=9$

e per $a=9$ e $b=4$

④ $\gamma: y = \ln x \quad 1 \le x \le 2\sqrt{2} \quad \delta(x,y) = x^2$
nel piano $\quad m = ?$

$$m = \int_\gamma x^2 \, ds$$

$$y' = \frac{1}{x} \quad 1 + y'^2 = 1 + \frac{1}{x^2} = \frac{x^2+1}{x^2}$$

$$ds = \sqrt{\frac{x^2+1}{x^2}}\,dx = \frac{\sqrt{x^2+1}}{|x|}\,dx = \frac{\sqrt{x^2+1}}{x}\,dx$$

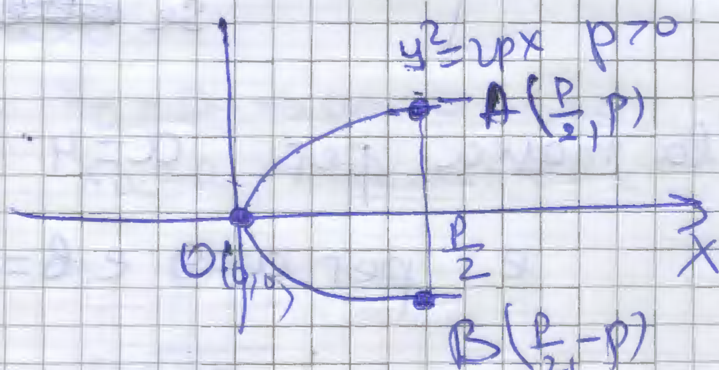
perché $x > 0$

$$\Rightarrow m = \int_1^{2\sqrt{2}} x^2 \cdot \frac{\sqrt{x^2+1}}{x}\,dx = \int_1^{2\sqrt{2}} x \cdot \sqrt{x^2+1}\,dx =$$

$$= \frac{1}{2}\int_1^{2\sqrt{2}} (x^2+1)^{\frac{1}{2}} d(x^2+1) = \frac{1}{2} \cdot \frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} \Bigg|_1^{2\sqrt{2}} =$$

$$= \frac{1}{3}\left[9^{3/2} - 2^{3/2}\right] = \frac{\sqrt{9^3} - \sqrt{2^3}}{3} = \boxed{\frac{27 - 2\sqrt{2}}{3}}$$

⑤ γ: è la parabola $y^2 = 2px \quad 0 \le x \le \frac{p}{2}$
$p > 0$; $\delta(x,y) = |y|$



$x=0 \Rightarrow y^2=0 \Rightarrow y=0 \rightarrow O(0,0)$

$x=\frac{p}{2} \quad y^2 = 2p\cdot\frac{p}{2} = p^2 \Rightarrow y=\pm p$

$\Rightarrow A\left(\frac{p}{2}, p\right)$ e $B\left(\frac{p}{2}, -p\right)$

$$\Rightarrow \left| \begin{array}{l} x = \dfrac{t^2}{2p} \\ y = t \end{array} \right.$$

$-p \le t \le p$

$$m = \int_\gamma |y|\, ds$$

$|y| = |t|$

$\dot{x} = \frac{t}{p} \quad \dot{y} = 1 \quad ds = \sqrt{1+\frac{t^2}{p^2}}\,dt = \frac{\sqrt{t^2+p^2}}{p}\,dt$

$$m = \int_{-p}^{p} |t| \cdot \frac{\sqrt{t^2+p^2}}{p}\,dt = \frac{2}{p}\int_0^p t\cdot\sqrt{t^2+p^2}\,dt =$$

$$= \frac{2}{2p}\int_0^p (t^2+p^2)^{\frac{1}{2}}\,d(t^2+p^2) = \frac{1}{p}\cdot \left.\frac{(t^2+p^2)^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^p =$$

$$= \frac{2}{3p}\left[(2p^2)^{3/2} - (p^2)^{3/2}\right] = \frac{2\cdot\left(\sqrt{8p^6} - \sqrt{p^6}\right)}{3p} =$$

$$= \frac{2\left(2\sqrt{2}p^3 - p^3\right)}{3p} = \frac{2p^3\left(2\sqrt{2}-1\right)}{3p} = \frac{2\left(2\sqrt{2}-1\right)p^2}{3}$$

$\boxed{\sqrt{p^6} = |p|^3 = p^3 \text{ perché } p>0}$

(6) $\gamma: \begin{cases} x = a\cos t \\ y = a\sin t \\ z = ct \end{cases}$ $\quad 0 \le t \le 2\pi$

$\boxed{a > 0}$ $\quad \delta(x,y) = x^2+y^2 \quad m = ?$

$$m = \int_\gamma (x^2+y^2)\,ds$$

$x^2+y^2 = a^2\cos^2 t + a^2\sin^2 t = a^2 =$ costante

$$\Rightarrow m = a^2 \cdot L_\gamma = a^2 \cdot \int_\gamma ds$$

$\dot{x} = -a\sin t, \quad \dot{y} = a\cos t \quad \dot{z} = c$

$$ds = \sqrt{a^2\sin^2 t + a^2\cos^2 t + c^2}\,dt = \sqrt{a^2+c^2}\,dt$$

$$\Rightarrow m = a^2 \int_0^{2\pi} \sqrt{a^2+c^2}\,dt = a^2 \cdot \sqrt{a^2+c^2} \cdot t \Big|_0^{2\pi} =$$

$$= \boxed{2a^2 \cdot \sqrt{a^2+c^2} \cdot \pi}$$

(7) $\gamma: \begin{cases} x = at \\ y = \frac{a}{2}t^2 \\ z = \frac{a}{3}t^3 \end{cases}$ fra le p. $O(0,0,0)$ e p. $A(a, \frac{a}{2}, \frac{a}{3})$

$a > 0$ $\quad \delta(x,y,z) = \sqrt{\frac{2y}{a}}$

$x = at \quad x_O = 0 \Rightarrow at = 0 \quad t = 0$

$\quad x_A = a \Rightarrow at = a \quad t = 1$

$$\Rightarrow 0 \le t \le 1$$

$$m = \int_\gamma \sqrt{\frac{2y}{a}}\,ds$$

$$\sqrt{\frac{2y}{a}} = \sqrt{\frac{2}{a}\cdot\frac{a}{2}t^2} = \sqrt{t^2} = |t| = t$$

perchè $t \geq 0$

$$\dot{x} = a \quad \dot{y} = at \quad \dot{z} = at^2$$

$$ds = \sqrt{a^2 + a^2t^2 + a^2t^4}\,dt = a \cdot \sqrt{t^4 + t^2 + 1}\,dt$$

$$m = a\int_0^1 t \cdot \sqrt{t^4+t^2+1}\,dt = \frac{a}{2}\int_0^1 \sqrt{t^4+t^2+1}\,d(t^2) =$$

Poniamo $t^2 = x$; $t = 0 \Rightarrow x = 0$
$t = 1 \Rightarrow x = 1$

$$= \frac{a}{2}\int_0^1 \sqrt{x^2+x+1}\,dx$$

$\boxed{x = z - \frac{1}{2}}$ $\boxed{dx = dz}$

$z = x + \frac{1}{2}$ $\quad x = 0 \Rightarrow z = 1/2$
$x = 1 \Rightarrow z = 3/2$

$$\sqrt{x^2+x+1} = \sqrt{z^2 - z + \frac{1}{4} + z - \frac{1}{2} + 1} = \sqrt{z^2 + \frac{3}{2}}$$

$$= \frac{a}{2}\int_{1/2}^{3/2} \sqrt{z^2 + \frac{3}{2}}\,dz \quad \text{per parti.}$$

(8) $\begin{cases} x = t \\ y = \frac{t^2}{2} \\ z = \frac{t^3}{3} \end{cases}$ $\quad 0 \leq t \leq 1 \quad \delta(x,y,z) = \sqrt{2y}$

$m = ?$