REMARKS ON ULTRAFILTERS ON THE COLLECTION OF FINITE SUBSETS OF AN INFINITE SET

ARTHUR D. GRAINGER

Let J be an infinite set and let $I = \mathcal{P}_f(J)$, i.e., I is the collection of all non empty finite subsets of J. Let βI denote the collection of all ultrafilters on the set I. In this presentation, we consider $(\beta I, \uplus)$, the compact (Hausdorff) right topological semigroup that is the *Stone−Čech Compactification* of the semigroup $(I, \cup)$ equipped with the discrete topology. We show that there is an injective map $A \to \beta_A(I)$ of $\mathcal{P}(J)$ into $\mathcal{P}(\beta I)$ such that each $\beta_A(I)$ is a closed subsemigroup of $(\beta I, \uplus)$, the set $\beta_J(I)$ is a closed ideal of $(\beta I, \uplus)$ and the collection $\{\beta_A(I) \mid A \in \mathcal{P}(J)\}$ is a partition of βI. This map is defined as follows. For $j \in J$, let $\mathring{j} = \{i \in I \mid j \in i\}$ and for $A \subseteq J$, let $\mathcal{G}_A = \left\{ \mathring{j} \mid j \in A \right\} \bigcup \left\{ I \backslash \mathring{j} \mid j \in J \backslash A \right\}$ [where $Y \backslash X = \{y \in Y \mid y \notin X\}$ for sets X and Y]. So, for $A \subseteq J$, define $\beta_A(I) = \{ p \in \beta I \mid \mathcal{G}_A \subseteq p \}$. Also, we show that for $A \subseteq J$ and $\mathcal{V}_A = \bigcup \{\beta_B(I) \mid B \subseteq A\}$, $(\mathcal{V}_A, \uplus)$ is a compact subsemigroup of $(\beta I, \uplus)$, $\mathcal{V}_A$ is the largest subsemigroup that has $\beta_A(I)$ as an ideal and $\beta_A(I)$ is the smallest ideal of $\mathcal{V}_A$ *if and only if* the complement of A (in J) is finite.

Department of Mathematics, Morgan State University, Baltimore, Maryland 21251 U.S.A.

E-mail address: `arthur.grainger@morgan.edu`