# TOPOLOGY FROM A REMOTE POINT OF VIEW

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We want to present some properties of topological spaces in terms of neighbourhood-monads, especially using so-called remote points. Before this we recall that by some Saturation-Principle (or Idealisation) the *filtermonad* $\bigcap_{F\in\mathcal{F}} {}^*F$ of every filter $\mathcal{F}$ is not empty.

Given a topological space $(X, \mathcal{T})$ and the $*$-image $({}^*X, {}^*\mathcal{T})$ in some nonstandard enlargement like an ultraproduct (or beginning with a standard topological space in some setting like **IST**) for every subset $\mathfrak{A} \subseteq {}^*X$ of ${}^*X$ the family $\mathcal{F}(\mathfrak{A}) := \{V \in \mathcal{T} : \mathfrak{A} \subseteq {}^*V\}$ defines a filter-base in $X$. So the *neighbourhood-monad* $\mu_{\mathcal{T}}(\mathfrak{A}) := \bigcap_{V\in\mathcal{F}(\mathfrak{A})} {}^*V \neq \emptyset$ is not empty (of course, it contains $\mathfrak{A}$).

We call an (internal) element $\mathfrak{x} \in {}^*X$ *near-standard* if there exists some $x \in X$ with $\mathfrak{x} \in \mu_{\mathcal{T}}({}^*x)$ and *remote* otherwise. $\mathtt{ns}({}^*X)$ be the set of all near-standard elements and $\mathtt{rmt}({}^*X) := {}^*X \setminus \mathtt{ns}({}^*X)$ the set of all remote points. Is $\mathcal{F}_d(\mathfrak{A}) = \{M \subseteq X : \mathfrak{A} \subseteq {}^*M\}$ the *discrete filter* and $\delta(\mathfrak{A})$ its filtermonad, we can easily show that $\mathfrak{x} \in \mathtt{rmt}({}^*X) \iff \delta(\{\mathfrak{x}\}) \subseteq \mathtt{rmt}({}^*X)$: Because for any $\mathfrak{x} \in {}^*X$ nonstandard, $\mathcal{F}_d(\{\mathfrak{x}\})$ is an ultrafilter it follows from $\delta(\{\mathfrak{x}\}) \cap \mu_{\mathcal{T}}({}^*x) \neq \emptyset$ for $x \in X$ that $\delta(\{\mathfrak{x}\}) \subseteq \mu_{\mathcal{T}}({}^*x)$.

It is very well known that we can talk about topological notions and properties such as open and closed sets, neighbourhoods, closures, continuity and so on in terms of neighbourhood-monads of standard points (of the form ${}^*x$). But more interesting is the fact, that there is much information about the topology hidden in the behaviour of remote points. For example regularity and normality, or things like locally finite families and paracompactness can be discussed using especially remote points and neighbourhood-monads of remote points.

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