ULTRAFILTERS IN MEASURE THEORY

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An ultrafilter $\mathcal{F}$ on $\mathbb{N}$ is **measure-centering** if whenever $\langle K_n\rangle_{n\in\mathbb{N}}$ is a sequence of compact subsets of $[0,1]$ such that $\lim_{n\to\mathcal{F}}\mu_L K_n > 0$, where μ_L is Lebesgue measure, then there is an $A \in \mathcal{F}$ such that $\bigcap_{n\in A} K_n \neq \emptyset$. For any such ultrafilter, and for any perfect probability space (X,μ), there is a measure $\mu_{\mathcal{F}}$ on X, extending μ, such that $\mu_{\mathcal{F}}(\lim_{n\to\mathcal{F}} E_n)$ is defined and equal to $\lim_{n\to\mathcal{F}}\mu E_n$ for every sequence $\langle E_n\rangle_{n\in\mathbb{N}}$ of sets measured by μ. I will describe the measure algebras and function spaces of such extensions, and sketch the proof of M. Benedikt's theorem that there is a common extension of the measures $\mu_{\mathcal{F}}$ as $\mathcal{F}$ runs over the family of all Ramsey ultrafilters.

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