

Corso di Laurea in Ingegneria dell'Energia

ANALISI MATEMATICA I

ESERCIZI SULLE EQUAZIONI DIFFERENZIALI

Risolvere le seguenti equazioni differenziali del primo ordine:

$$(1) \quad u'(t) + u(t) \sin t = \sin t;$$

$$(2) \quad (1 + t^2)u'(t) + 2t u(t) = t^3;$$

$$(3) \quad u'(t) = 2t[t^2 + u(t)].$$

Risolvere i seguenti problemi di Cauchy:

$$(4) \quad \begin{cases} u'(t) - 2t u(t) = t \\ u(0) = 0 \end{cases}$$

$$(5) \quad \begin{cases} u'(t) - e^t u(t) = e^t \\ u(1) = e^e - 1 \end{cases}$$

$$(6) \quad \begin{cases} u'(t) = \sqrt{1 - u^2(t)} \quad t \\ u(0) = \frac{1}{2} \end{cases}$$

Risolvere le seguenti equazioni differenziali:

$$(7) \quad u''(t) - 2u'(t) - 15u(t) = 0;$$

$$(8) \quad u''(t) + (1 - i)u'(t) + iu(t) = 0;$$

$$(9) \quad u''(t) + 25u(t) = 20t^3 + 25t^5;$$

$$(10) \quad u''(t) - u'(t) - 30u(t) = -3 - 90t;$$

$$(11) \quad u''(t) - 3u(t) = e^{2t} - 7 \sin 2t;$$

$$(12) \quad u''(t) - 10u'(t) + 25u(t) = (6t + 2)e^{5t}.$$

RISPOSTE.

(1) $u(t) = C e^{\cos t} + 1;$

(2) $u(t) = \frac{t^4}{4(1+t^2)} + \frac{C}{1+t^2};$

(3) $u(t) = C e^{t^2} - (t^2 + 1);$

(4) $u(t) = \frac{e^{t^2} - 1}{2};$

(5) $u(t) = e^{e^t} - 1;$

(6) $u(t) = \sin\left(\frac{1}{2}t^2 + \frac{\pi}{6}\right);$

(7) $V_0 = \{C_1 e^{3t} + C_2 e^{-5t}, C_1, C_2 \in \mathbb{C}\};$

(8) $V_0 = \{C_1 e^{it} + C_2 e^t, C_1, C_2 \in \mathbb{C}\};$

(9) $V_f = \{C_1 e^{5it} + C_2 e^{-5it} + t^5, C_1, C_2 \in \mathbb{C}\} = \{C_1 \sin 5t + C_2 \cos 5t + t^5, C_1, C_2 \in \mathbb{C}\}$

(10) $V_f = \{C_1 e^{-5t} + C_2 e^{6t} + 3t, C_1, C_2 \in \mathbb{C}\}$

(11) $V_f = \{C_1 e^{\sqrt{3}t} + C_2 e^{-\sqrt{3}t} + e^t + \sin 2t, C_1, C_2 \in \mathbb{C}\};$

(12) $V_f = \{C_1 e^{5t} + C_2 t e^{5t} + e^{5t}(t^2 + t^3), C_1, C_2 \in \mathbb{C}\}.$