

Calcolo di limiti  
con il principio di sostituzione

$$1. \lim_{x \rightarrow 0} \frac{\sin x + 1 - \cos x}{e^x - 1 - \lg(1+x^2)} = -1$$

$$\sin x \sim x \quad e^x - 1 \sim x$$

$$1 - \cos x \sim \frac{1}{2}x^2 \quad \lg(1+x^2) \sim x^2$$

$$f(x) \sim \frac{x}{x} \rightarrow 1$$

$$2. \lim_{x \rightarrow 0} \frac{\sin^2 x + 1 - \cos x}{(1+x^2)^3 - 1} = \frac{1}{2}$$

$$\sin^2 x \sim x^2 \quad 1 - \cos x \sim \frac{1}{2}x^2$$

$$(1+x^2)^3 - 1 \sim 3x^2$$

$$f(x) \sim \frac{\frac{3}{2}x^2}{3x^2} \rightarrow \frac{1}{2}$$

$$3. \lim_{x \rightarrow 0} \frac{\sin x + \operatorname{tg} x + x^2}{e^{2x} - \sqrt{1+x}} = \frac{4}{3}$$

$$\sin x \sim x \quad \operatorname{tg} x \sim x$$

$$e^{2x} \sim 1 + 2x \quad \sqrt{1+x} \sim 1 + \frac{1}{2}x$$

$$f(x) \sim \frac{2x}{\frac{1}{2}x} \rightarrow \frac{4}{3}$$

$$4. \lim_{x \rightarrow 0} \frac{(4 - e^x) \sin x + \operatorname{tg} x + \lg(1 + \sin^2 x)}{(\sqrt{1+x} - 1)(\sqrt[4]{1+x} + 1)} = 4$$

$$(4 - e^x) \sin x \sim 3x \quad \operatorname{tg} x \sim x \quad \lg(1 + \sin^2 x) \sim x^2$$

$$\sqrt{1+x} - 1 \sim \frac{1}{2}x \quad \sqrt[4]{1+x} + 1 \sim 2$$

$$f(x) \sim \frac{4x}{x} \rightarrow 4$$

$$5. \quad \lim_{x \rightarrow +\infty} x(e^{1/\sqrt{x}} - \cos \frac{1}{\sqrt{x}}) = \frac{3}{2}$$

$$e^{1/\sqrt{x}} \sim 1 + \frac{1}{\sqrt{x}} \quad \cos \frac{1}{\sqrt{x}} \sim 1 - \frac{1}{2x}$$

$$f(x) \sim x \cdot \frac{3}{2x} \rightarrow \frac{3}{2}$$

$$\lim_{x \rightarrow 0^+} \frac{(5x+1)^{10} - 1}{(e^{\sqrt{x}} - 1) \sin \sqrt{x}} = 50$$

$$(5x+1)^{10} - 1 \sim 50x$$

$$e^{\sqrt{x}} - 1 \sim \sqrt{x} \quad \sin \sqrt{x} \sim \sqrt{x}$$

$$f(x) \sim \frac{50x}{x} \rightarrow 50$$

$$\lim_{x \rightarrow +\infty} \frac{x^4 + x^3 - 1}{x^3 + 3 + \lg x} = +\infty$$

$$\begin{aligned} x^4 + x^3 - 1 &\sim x^4 \\ x^3 + 3 + \lg x &\sim x^3 \end{aligned}$$

$$f(x) \sim x^4/x^3 = x \rightarrow +\infty$$

$$\lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x}\right) \lg x - \lg x^3 = -\infty$$

$$\left(2 - \frac{1}{x}\right) \lg x \sim 2 \lg x$$

$$\lg x^3 = 3 \lg x$$

$$f(x) \sim - \lg x$$

$$\lim_{x \rightarrow +\infty} e^x - x^3 + 3 \sin x = +\infty$$

$$e^x - x^3 + 3 \sin x \sim e^x \rightarrow +\infty$$

$$10. \quad \lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x}\right) \lg^3 x - 3^x = -\infty$$

$$\left(2 - \frac{1}{x}\right) \lg^3 x \sim 2 \lg^3 x$$

$$f(x) \sim 2 \lg^3 x - 3^x \sim -3^x \rightarrow -\infty$$

$$11. \quad \lim_{x \rightarrow +\infty} \sqrt{x^3 + x + 1} - 2x\sqrt{x} = -\infty$$

$$f(x) \sim x\sqrt{x} - 2x\sqrt{x} = -x\sqrt{x} \rightarrow -\infty$$

$$12. \quad \lim_{x \rightarrow +\infty} \frac{x^2 - x \lg^4 x + \cos x}{\sqrt{1+x^2} + \lg^4 x} = +\infty$$

$$f(x) \sim \frac{x^2}{x} = x \rightarrow +\infty$$

$$13. \quad \lim_{x \rightarrow -\infty} \frac{5x+7}{\sqrt{x^2+4}} = -5$$

$$f(x) \sim \frac{5x}{-x} \rightarrow -5$$

$$14. \quad \lim_{x \rightarrow -\infty} x^2 e^x = 0$$

$$t = -x \rightarrow +\infty \quad t^2 e^{-t} = \frac{t^2}{e^t} \rightarrow 0$$

15. Ordinare in ordine crescente di infinito per  $x \rightarrow +\infty$  :

$$e^{x^3} \quad e^{3x} \quad e^{x^3 \lg x} \quad e^{(\lg x)^2} \quad x^5 (\lg x)^2 \quad x^x$$

Riscriviamo tutte le fz. in forma esponenziale :

$$\begin{aligned} x^5 (\lg x)^2 &= e^{\lg(x^5 (\lg x)^2)} = e^{\lg x^5 + \lg (\lg x)^2} \\ &= e^{5 \lg x + 2 \lg \lg x} \sim e^{5 \lg x} \end{aligned}$$

$$x^x = e^{x \lg x}$$

Confrontiamo gli esponenti :

$$x^3 \quad 3x \quad x^3 \lg x \quad (\lg x)^2 \quad 5 \lg x \quad x$$

Ordiniamoli in modo crescente di infinito.

$$5 \lg x \quad (\lg x)^2 \quad x \quad 3x \quad x^3 \quad x^3 \lg x.$$