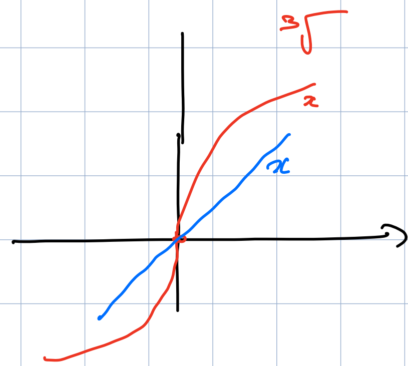
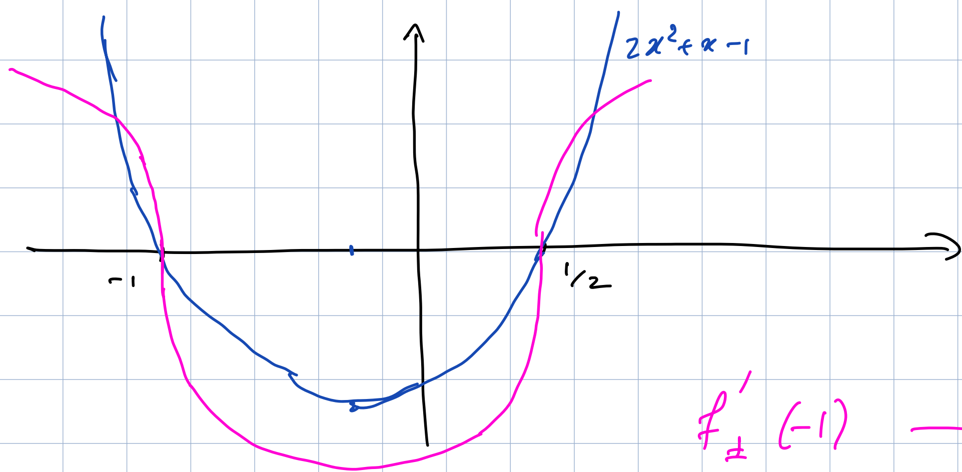
Ist. Mat. I - CIA
22/11/23

p. 171   Trovare $f'_\pm$ dove esiste o no ...

(34) $\sqrt[3]{2x^2+x-1}$

$= \sqrt[3]{(2x-1)(x+1)}$





$f'_\pm(-1) \to -\infty$

$f'_\pm(+1/2) \to +\infty$

Ese: min $x = -1/4$

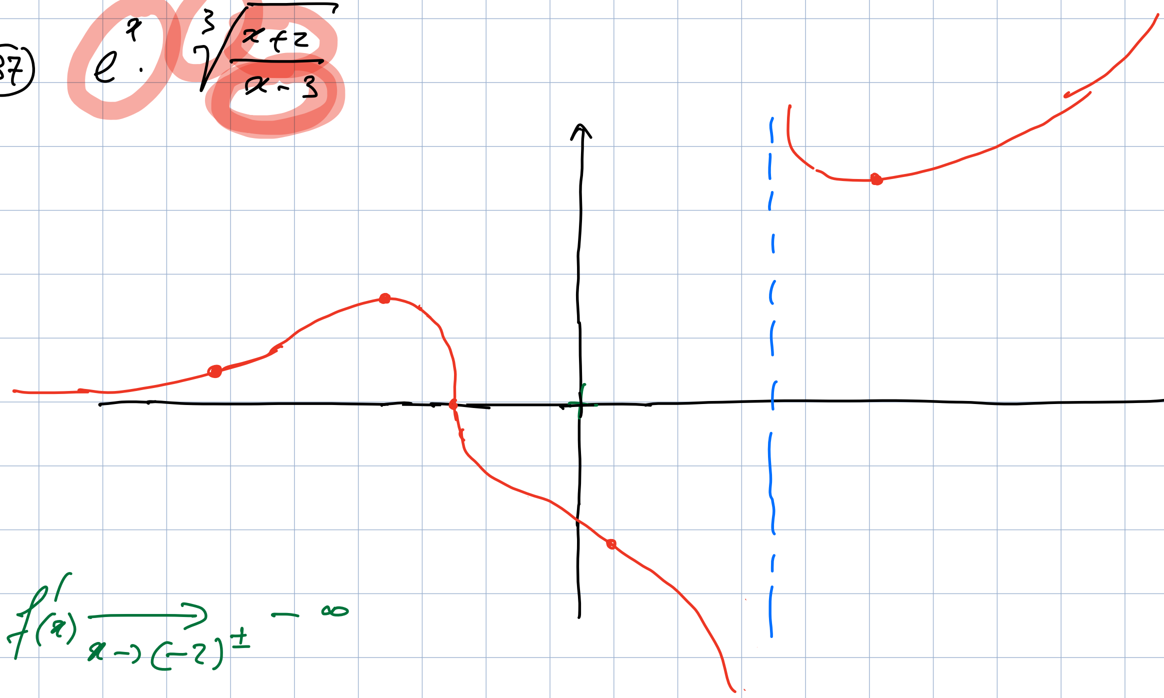
conc su $(-\infty, -1]$ e $[1/2, +\infty)$

conv. su $[-1, 1/2]$

(35) $e^{-|x|}$

$f'_\pm(0) = \pm 1$

(37) $e^x \cdot \sqrt[3]{\dfrac{x+2}{x-3}}$

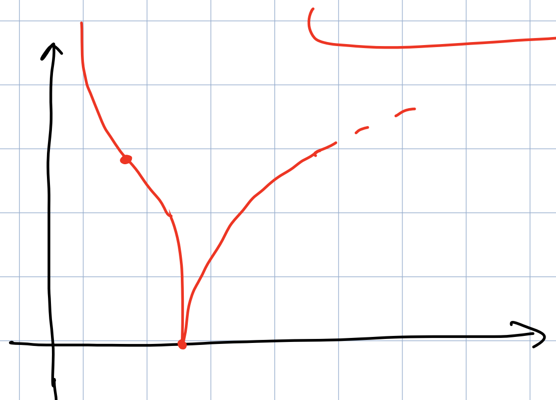


$f'(x) \xrightarrow[x \to (-2)^{\pm}]{} -\infty$

Ex: trovare pti di max/min loc + flessi

(38) $\log^2\left(1+\sqrt[3]{x}\right)$ $\qquad 1+\sqrt[3]{x} > 0, \quad \sqrt[3]{x} > -1, \quad x > -1$

$x \neq 0 \qquad f'(x) = \underbrace{2 \cdot \underbrace{\log\left(1+\sqrt[3]{x}\right)}_{0^{\pm}} \cdot \underbrace{\frac{1}{1+\sqrt[3]{x}}}_{1} \cdot \frac{1}{3} \cdot \underbrace{\frac{1}{\sqrt[3]{x^2}}}_{+\infty}}_{\pm\infty}$



Es: verificare no max/min
Trovare flesso.

(39) $$f(x) = \begin{cases} x^2 \cdot \sin\left(\frac{1}{x}\right) & x > 0 \\ 0 & x \le 0 \end{cases}$$

$\exists f'$? Continua?

$\exists f'(x) \quad \forall x \neq 0$

$f'(x) = 0 \quad \forall x < 0 \qquad f'_-(0) = 0$

$x > 0 \qquad f'(x) = 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right)$

$$= \underbrace{2x \cdot \sin\left(\frac{1}{x}\right)}_{\downarrow\, 0} - \underbrace{\cos\left(\frac{1}{x}\right)}_{\nexists \lim}$$

$$f'_+(0) = \lim_{h \to 0} \frac{h^2 \cdot \sin\left(\frac{1}{h}\right) - 0}{h} = \lim h \cdot \sin\left(\frac{1}{h}\right) = 0$$

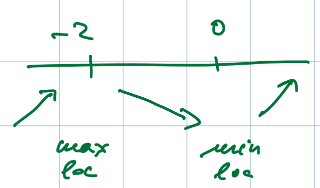
$\Rightarrow \exists f'(x) \quad \forall x \in \mathbb{R}$ discont. in 0.

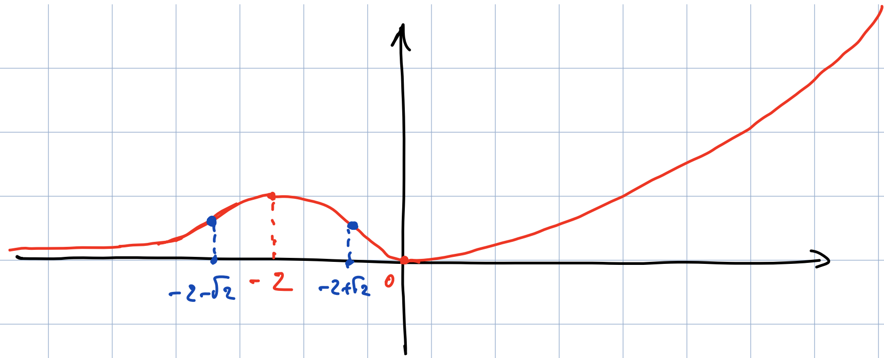
pag. 193. Trovare max/min e disegnare grafico

D = più grande su cui $f(x)$ ha senso.

(41) $x^2 \cdot e^x \qquad D = \mathbb{R} \qquad \lim_{+\infty} = +\infty \qquad \lim_{-\infty} = 0$

$f'(x) = 2x \cdot e^x + x^2 \cdot e^x = x(2+x) \cdot e^x$





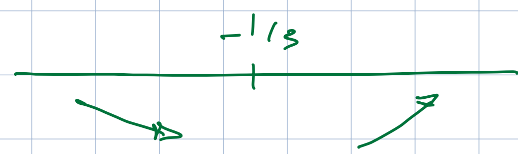
$$f''(x) = (2+2x)\cdot e^x + (2x+x^2)\cdot e^x$$
$$= (x^2+4x+2)\cdot e^x$$

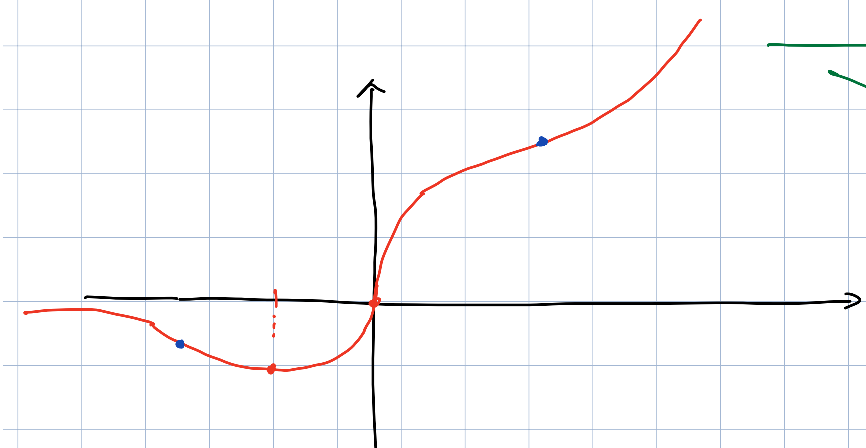
$$-2 \pm \sqrt{2}$$

(42) $\sqrt[3]{x}\cdot e^x$ $\qquad \mathbb{R}$

$\lim_{-\infty} = 0^-$ $\qquad \lim_{+\infty} = +\infty$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}\cdot e^x + \sqrt[3]{x}\cdot e^x = \frac{1}{\sqrt[3]{x^2}}\left(\frac{1}{3} + x\right)e^x$$



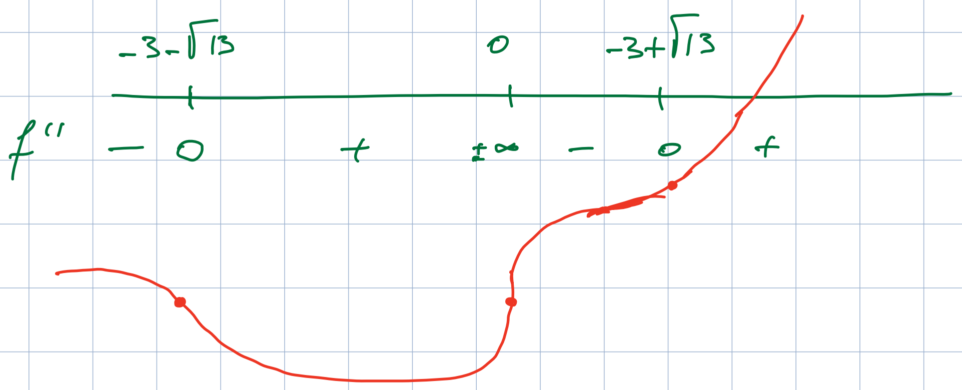


$$f'(x) = \left(\frac{1}{3}x^{-\frac{2}{3}} + x^{\frac{1}{3}}\right)\cdot e^x$$

$$f''(x) = \left(-\frac{4}{9}x^{-\frac{5}{3}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{3}x^{-\frac{2}{3}} + x^{\frac{1}{3}}\right) \cdot e^{x}$$

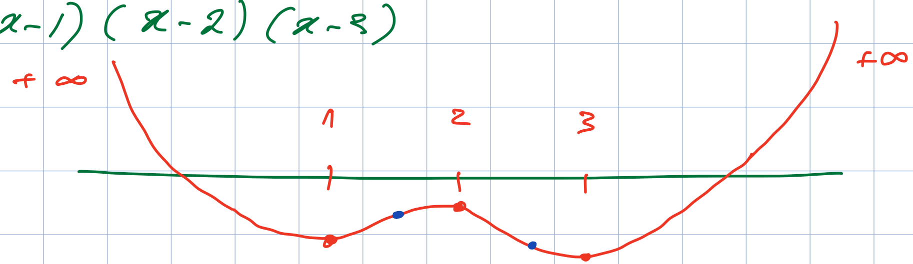
$$= \frac{1}{9x^{5/3}} \cdot \left(-4 + 6x + x^2\right) \cdot e^{x}$$

$$-3 \pm \sqrt{9+4} = -3 \pm \sqrt{13}$$



(44) $x^4 - 8x^3 + 22x^2 - 24x + 12$

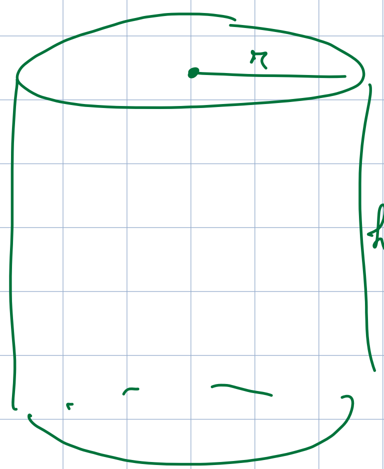
$$f'(x) = 4x^3 - 24x^2 + 44x - 24$$
$$= 4\left(x^3 - 6x^2 + 11x - 6\right)$$
$$= 4(x-1)\left(x^2 - 5x + 6\right)$$
$$= 4(x-1)(x-2)(x-3)$$



$$f''(x) = 4\left(3x^2 - 12x + 11\right)$$

$$\frac{6 \pm \sqrt{36-33}}{3} = \frac{6 \pm \sqrt{3}}{3} = 2 \pm \frac{1}{\sqrt{3}}$$

(49) Qual è la lattina da 33 cl di minimo peso.



Uso cm come unità

$$Vol = 330 = \pi r^2 h$$

$$\Rightarrow h = \frac{330}{\pi r^2}$$

$$Peso = \underline{superficie \times \dots}$$

$$= 2\pi r^2 + 2\pi r \cdot h$$

$$= 2\pi r^2 + 2\pi r \cdot \frac{330}{\pi r^2}$$

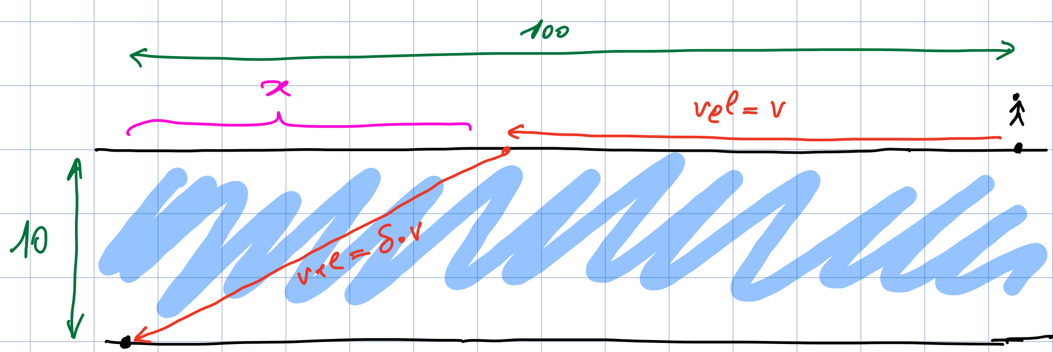
$$= 2\pi r^2 + \frac{660}{r}$$

Deriv: $$4\pi r - 660 \cdot \frac{1}{r^2} = 0$$

$$r^3 = \frac{660}{4\pi}$$

$$r = \sqrt[3]{\frac{165}{\pi}} = 3.74\dots$$

(50)



$\delta < 1$

Dove gli conviene tuffarsi per fare prima.

Oss: $\delta = 1$ si tuffa subito.

Per quali δ si tuffa subito.

$$t = \frac{100-x}{v} + \frac{\sqrt{x^2+100}}{\delta \cdot v} = \cancel{\frac{1}{v}} \cdot \left(100 - x + \frac{\sqrt{x^2+100}}{\delta}\right)$$

$$t' = -1 + \frac{1}{\delta} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x^2+100}} \cdot 2x$$

$$= -1 + \frac{x}{\delta \cdot \sqrt{x^2+100}}$$

$t' = 0$

$$x = \delta \sqrt{x^2+100}$$

$$x^2 = \delta^2 x^2 + \delta^2 \cdot 100$$

$$(1-\delta^2)x^2 = \delta^2 \cdot 100$$

$$x = \frac{10\delta}{\sqrt{1-\delta^2}}.$$

$$\lim_{\delta \to 1^-} x = +\infty$$

Si tuffa con $x = \frac{10\delta}{\sqrt{1-\delta^2}}$ se ciò è ≤ 100

Si tuffa subito se

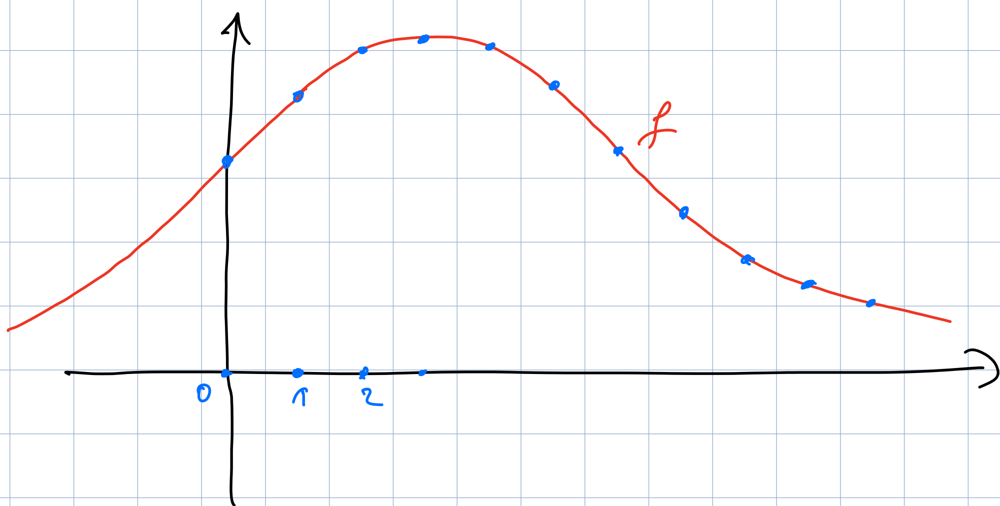
$$\frac{10\delta}{\sqrt{1-\delta^2}} \geq 100 \qquad \frac{\delta}{\sqrt{1-\delta^2}} \geq 10$$

$$\delta^2 \geq 100(1-\delta^2) \qquad 101\delta^2 \geq 100$$

$$\delta \geq \frac{10}{\sqrt{101}} = 0.995\ldots$$

(54) Dire se (a_n) è monotona da un certo punto in poi.

- $\frac{n+3}{n^2-2n+4} = f(n)$ $\qquad f(x) = \frac{x+3}{x^2-2x+4}$



$$f'(x) = \frac{1\cdot(x^2-2x+4)-(x+3)(2x-2)}{(\quad)^2}$$

$$= \frac{x^2-2x+4-2x^2+2x-6x+6}{(\quad)^2}$$

$$= \frac{-x^2-6x+10}{(\quad)^2}$$ negativo per $x \geq 2$

SÌ

$n^2 \cdot \log(n)$ $\qquad 2x\cdot\log(x) + x^2\cdot\frac{1}{x} = x(2\log(x)+1)$

pos. per $x \geq 1$

SÌ

$\dfrac{n!}{2^n}$ Non è $f(n)$ con $f: [0,\infty) \to \mathbb{R}$.

$$a_{n+1} - a_n = \frac{(n+1)!}{2^{n+1}} - \frac{n!}{2^n} = \frac{n!}{2^{n+1}} \cdot (n+1-2)$$
$$= \frac{n!}{2^{n+1}} (n-1)$$

cresce da $n=1$ in poi

$1, \frac{1}{2}, \frac{24}{4} = 6, \nearrow \dots$

Calcolare i lim dati con de l'Hôpital se possibile.

$\left(\pi/2x \; \begin{matrix} \frac{\pi}{2} \cdot x \\ \frac{\pi}{2x} \end{matrix} \right)$

55 $$\lim_{x \to 1} \frac{\cos\left(\frac{\pi}{2x}\right)}{\sin(\pi x)}$$

$$x \to 1 \quad \frac{\cos\left(\frac{\pi}{2}\right)}{\sin(\pi)} = \frac{0}{0}$$

$$\text{dlH} \rightsquigarrow \frac{+\overbrace{\sin\left(\frac{\pi}{2x}\right)}^{1} \cdot \cancel{\frac{\pi}{2}} \cdot \overbrace{\left(+\frac{1}{x^2}\right)}^{1}}{\cancel{\pi} \cdot \underbrace{\cos(\pi x)}_{-1}} \longrightarrow -\frac{1}{2}$$

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$$\lim_{x \to \infty} \frac{x^x}{2^{x^2}} \qquad \frac{\infty^\infty}{2^{\infty^2}} = \frac{\infty}{\infty}$$

$$\text{dH} \rightsquigarrow \frac{(x^x)'}{(2^{x^2})'} = \frac{\left(e^{\log(x^x)}\right)'}{\left(e^{\log(2^{x^2})}\right)'} = \frac{\left(e^{x \cdot \log(x)}\right)'}{\left(e^{x^2 \cdot \log(2)}\right)'}$$

$$= \frac{x^x}{2^{x^2}} \cdot \frac{\log(x) + x \cdot \frac{1}{x}}{2x \cdot \log 2} = \frac{x^x \cdot (\log(x)+1)}{x \cdot 2^{x^2+1} \cdot \log(2)} = \frac{\infty}{\infty}$$

$$\frac{x^x}{2^{x^2}} = \frac{x^x}{2^{x \cdot x}} = \frac{x^x}{(2^x)^x} = \underbrace{\left(\underbrace{\frac{x}{2^x}}_{0^+}\right)^{\overbrace{x}^{+\infty}}}_{0^+}$$

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$$\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$$

Taylor $\qquad e^x = \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n) \qquad \forall n$

$$= 1 + x + \frac{1}{2} x^2 + o(x^2)$$

$$\frac{e^x - x - 1}{x^2} = \frac{\cancel{1+x}+\frac{1}{2}x^2 + o(x^2) \cancel{-1-x}}{x^2} = \frac{1}{2} + o(1) \to \frac{1}{2}$$

dlH : $\frac{0}{0} \rightsquigarrow \frac{e^x - 1}{2x} = \frac{0}{0} \rightsquigarrow \frac{e^x}{2} \longrightarrow \frac{1}{2}$

(58) $\lim\limits_{x \to 2^+} \frac{\sqrt{x^2-4}}{\sin(x-2)}$

dlH : $\frac{0}{0} \rightsquigarrow \frac{\frac{1}{\cancel{2}} \cdot \overbrace{\frac{1}{\sqrt{x^2-4}}}^{+\infty} \cdot \cancel{2}x \Big\}^{2}}{\underbrace{\cos(x-2)}_{1}} \longrightarrow +\infty$

A mano: $\frac{\sqrt{x-2} \cdot \sqrt{x+2}}{\sin(x-2)}$ $\qquad t = x-2$

$$= \frac{\sqrt{t} \cdot \sqrt{t+4}}{\sin(t)} = \underbrace{\underbrace{\frac{t}{\sin(t)}}_{\downarrow\, 1} \cdot \underbrace{\frac{1}{\sqrt{t}}}_{\downarrow\, +\infty} \cdot \underbrace{\sqrt{t+4}}_{\downarrow\, 2}}_{+\infty.} \qquad t \to 0^+$$