Geometria 20/5/22

13 (3) $\{[1-t : 3t : 7-t] : t \in \mathbb{R}\}$
$\{[1-t : 1+t : -2t] : t \in \mathbb{R}\}$

$$\cancel{(1-t,\ 3t,\ 7-t) = (1-t,\ 1+t,\ -2t)}$$

$$\cancel{[1-t : 3t : 7-t] = [1-t : 1+t : -2t]}$$

$$\cancel{(1-t,\ 3t,\ 7-t) = (1-s,\ 1+s,\ -2s)}$$

$$[1-t : 3t : 7-t] = [1-s : 1+s : -2s]$$

cioè

$$\begin{pmatrix} 1-t & 3t & 7-t \\ 1-s & 1+s & -2s \end{pmatrix}$$

ha rango 1, cioè tutti e tre i det 2x2 sono 0.

$(s = 1/5)$
...

$$+6st + (1+s)(7-t) = 0$$

$$t(6s - 1 - s) = -7(1+s) \qquad t = \frac{7(1+s)}{1-5s}$$

$$\left(1-\frac{7(1+s)}{1-5s}\right)\cancel{(1+s)}=(1-s)\frac{21\cancel{(1+s)}}{1-5s} \qquad (s=-1\ \ldots)$$

$$1-5s-7-7s=21-21s$$

$$9s=27$$

$$s=3 \qquad t=\frac{7\cdot 4}{-14}=-2$$

Sostituisco:

$$\begin{pmatrix} 1-t & 3t & 7-t \\ 1-s & 1+s & -2s \end{pmatrix} \to \begin{pmatrix} 3 & -6 & 9 \\ -2 & 4 & -6 \end{pmatrix}$$

ok : $\boxed{[1:-2:3]}$

Valori esclusi: $s=1/5$

$$\begin{matrix} 1-t & 3t & 7-t \\ \cancel{4} & \cancel{6} & \cancel{-2} \\ 2 & 3 & -1 \end{matrix} \qquad \begin{matrix} -3t=3(7-t) \\ 21=0 \quad \underline{\text{No}} \end{matrix}$$

$s=-1$

$$\begin{pmatrix} 1-t & 3t & 7-t \\ 2 & 0 & 2 \end{pmatrix}$$

④ $x^2+4kxy+ky^2+2x+y+3=0$ parabola per ...?

$$\begin{pmatrix} 1 & 2k & 1 \\ 2k & k & 1/2 \\ 1 & 1/2 & 3 \end{pmatrix}$$

$d_2 = k-4k^2 = k(1-4k)$

nullo per $k=0$, $k=1/4$

$\boxed{k=0}$ $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1/2 \\ 1 & 1/2 & 3 \end{pmatrix} \longrightarrow d_3 = -\frac{1}{4} \neq 0$ parabola

$k=1/4$ $\begin{pmatrix} 1 & 1/2 & 1 \\ 1/2 & 1/4 & 1/2 \\ 1 & 1/2 & 3 \end{pmatrix} \longrightarrow d_3 = \frac{3}{4}+\frac{1}{4}+\frac{1}{4}-\frac{1}{4}-\frac{3}{4}-\frac{1}{4}=0$ No

⑤ Tipo affine $x^2-4z^2+2xy-4yz+3x+5y+8z=0$

$$\begin{pmatrix} 1 & 1 & 0 & 3/2 \\ 1 & 0 & -2 & 5/2 \\ 0 & -2 & -4 & 4 \\ 3/2 & 5/2 & 4 & 0 \end{pmatrix}$$

$d_2 < 0$

$d_3 = 0+0+0+0+4-4=0$

$d_4 = \dots \neq 0$

parab. iperb.

⑦ $\int_\alpha 3xy^2dx + 2x^2ydy$ $\alpha(t) = (2t^2, t^3)$ su $[0,1]$

$\frac{\partial}{\partial x}(2x^2y) = 4xy \neq \frac{\partial}{\partial y}(3xy^2) = 6xy$

$$\int_0^1 \left(3\,(2t^2)(t^3)^2 \cdot 4t + 2 \cdot (2t^2)^2 (t^3)\, 3t^2\right) dt$$

$$= \int_0^1 \left(24\,t^9 + 24\,t^9\right) dt = \frac{48}{10}\, t^{10} \Big|_0^1 = \frac{24}{5}$$

$\boxed{7}$ (1) $\displaystyle\int_\alpha y \cdot \sin(xy^2)\,(y\,dx + 2x\,dy)$ $\qquad \alpha(t) = \left(t, \sin\left(\frac{t}{2}\right)\right)$

$t \in [0, \pi]$

$f\,dx + g\,dy \qquad f = \sin(xy^2) \cdot y^2$

$\qquad\qquad\qquad g = \sin(xy^2) \cdot 2xy$

$U(x,y) = -\cos(xy^2)$

$$\int_\alpha dU = -\cos(xy^2) \Big|_{(0,\dots)}^{(\pi,1)} = -\cos(\pi) + \cos(0) = 2$$

(2) $\displaystyle\int_\alpha x \qquad \alpha : [0,1] \to \mathbb{R}^2 \qquad \alpha(t) = (t, 2t^2)$

$$\int_\alpha \dots \;\to\; \int_\alpha F = \int_a^b F(\alpha(t)) \cdot \|\alpha'(t)\|\,dt$$

$$\int_\alpha \dots \;\to\; \int_\alpha f\,dx + g\,dy = \int_a^b f(\alpha(t)) \cdot x'(t) + g(\alpha(t)) \cdot y'(t)$$

$$\alpha'(t) = (1, 4t)$$

$$\int_0^1 t \cdot \sqrt{1+16t^2}\, dt = \frac{1}{32} \int_0^1 32t \sqrt{1+16t^2}\, dt$$

$$= \frac{1}{32} \cdot \frac{2}{3} \left(1+16t^2\right)^{3/2} \Big|_0^1$$

$$= \frac{1}{48} \left(17\sqrt{17} - 1 \right)$$

③ κ, τ in $s = 0$ $\qquad \alpha(s) = (2s^2, s^3, \sin(s))$

$$\alpha'(s) = \begin{pmatrix} 4s \\ 3s^2 \\ \cos(s) \end{pmatrix} \qquad \alpha''(s) = \begin{pmatrix} 4 \\ 6s \\ -\sin(s) \end{pmatrix} \qquad \alpha'''(s) = \begin{pmatrix} 0 \\ 6 \\ -\cos(s) \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix}$$

$$\kappa = \frac{\| \alpha' \wedge \alpha'' \|}{\| \alpha' \|^3} = \frac{\left\| \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\|^3} = 4$$

$$\tau = \frac{\langle \alpha' \wedge \alpha'' | \alpha''' \rangle}{\| \alpha' \wedge \alpha'' \|^2} = \frac{\left\langle \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \middle| \begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix} \right\rangle}{16} = \frac{24}{16} = \frac{3}{2}$$

(4) Intersezione tra

$$\{[2t-1 : t+1 : t-1] \;:\; t \in \mathbb{R}\} \subset \mathbb{P}^2(\mathbb{R})$$

con punti all'infinito di

$$\underbrace{2xy - y^2 + 3z^2}_{} - 7x + 2z = 3.$$

$$2(2t-1)(t+1) - (t+1)^2 + 3(t-1)^2 = 0$$

$$\underline{4t^2} + \cancel{4t} \cancel{-2t} \cancel{-2} \underline{-t^2} \cancel{-2t} \cancel{-1} \underline{+3t^2} \underline{-6t} \cancel{+3} = 0$$

$$6t^2 - 6t = 0$$

$t = 0 \qquad [-1 : 1 : -1] = [1 : -1 : 1]$

$t = 1 \qquad [1 : 2 : 0]$