Geometria 13/4/22

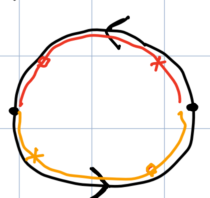
$$\mathbb{P}(V) = V\setminus\{0\} \Big/ {\scriptstyle v \sim w \text{ se } w = \lambda \cdot v} = \text{l'insieme delle rette in } V$$

$$\mathbb{P}^n(\mathbb{R}) = \mathbb{P}(\mathbb{R}^{n+1})$$

$$\mathbb{P}^0(\mathbb{R}) = \text{pto}$$

$$\mathbb{P}^n(\mathbb{R}) = S^n \Big/ {\scriptstyle x \sim -x} \qquad S^n = \{x \in \mathbb{R}^{n+1} : \|x\| = 1\}$$

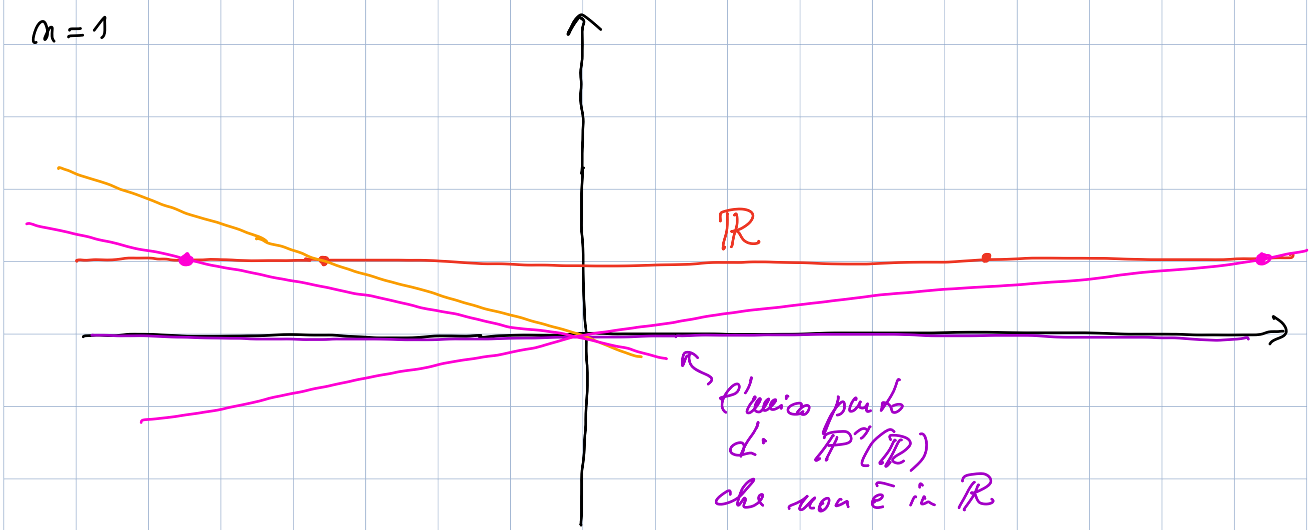
$$\mathbb{P}^0(\mathbb{R}) = \text{un pto}$$

$\mathbb{P}^1(\mathbb{R}) =$  $= S^1$

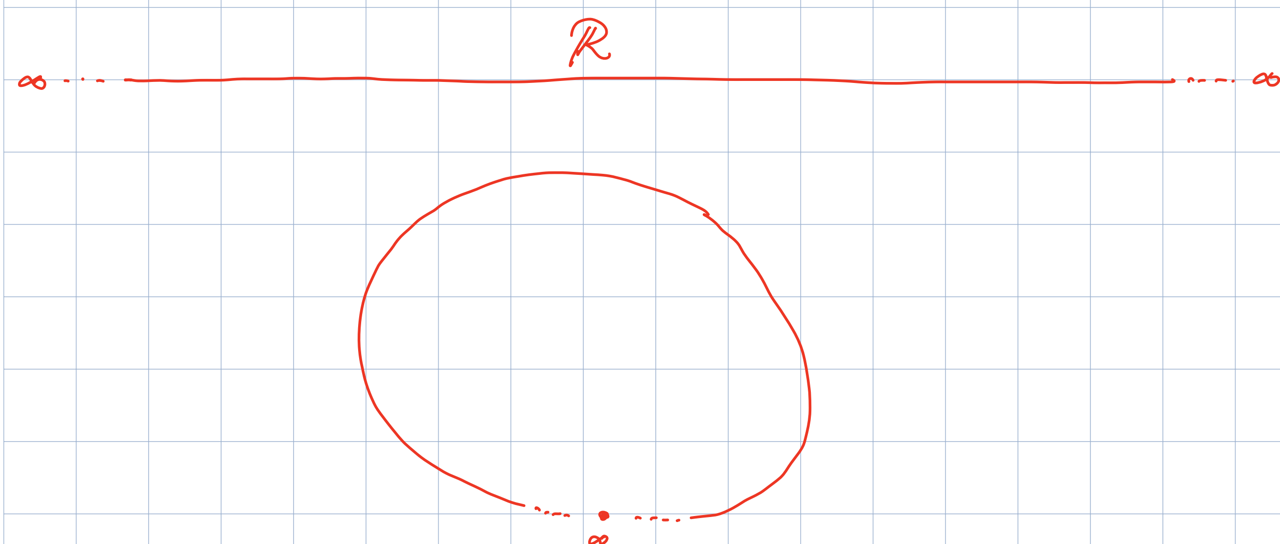
$$\mathbb{P}^n(\mathbb{R}) = \underset{\substack{\| \\ \mathbb{R}^n \times \{1\}}}{\mathbb{R}^n} \cup \underset{\substack{\| \\ \mathbb{P}(\mathbb{R}^n \times \{0\})}}{\mathbb{P}^{n-1}(\mathbb{R})}$$

i punti all'∞ di $\mathbb{R}^n$

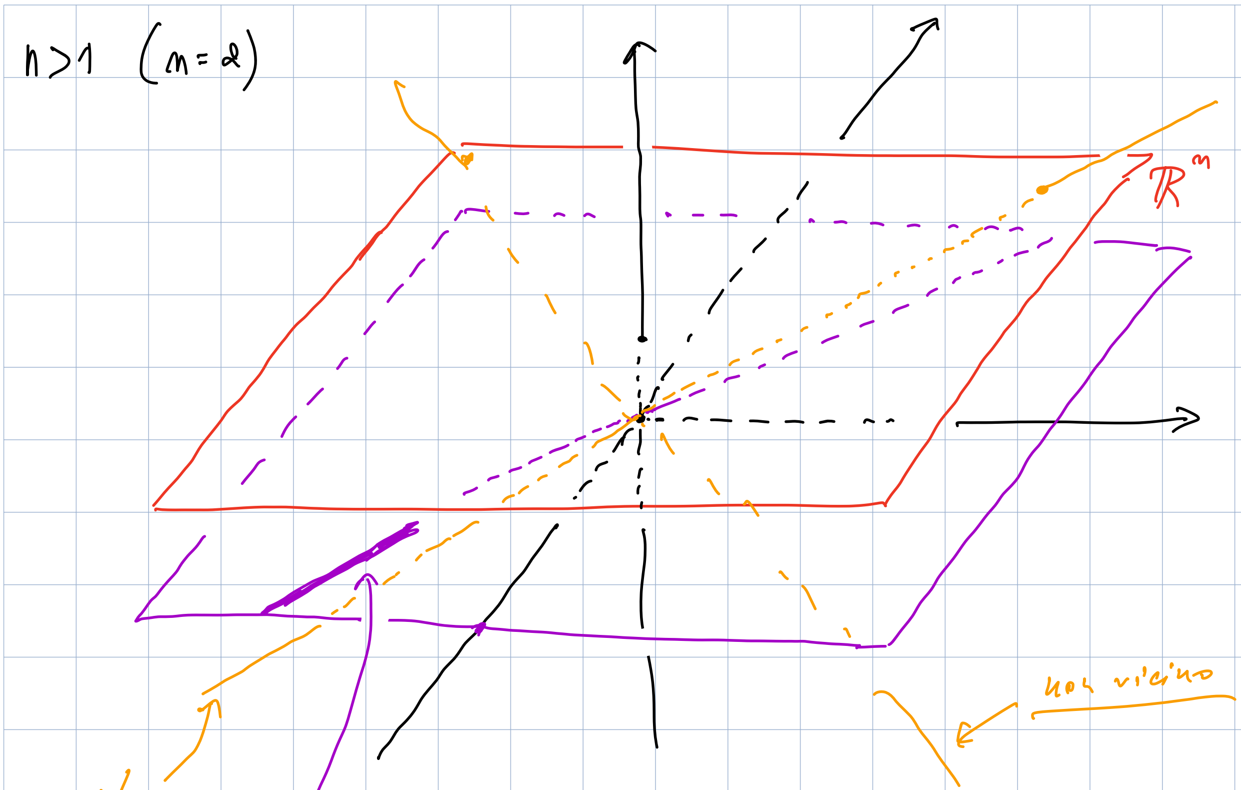
$n = 1$



rette vicine a quella viola = rette quasi orizzontali
= punti "molto lontani" da entrambe le parti



$n > 1 \quad (n = 2)$

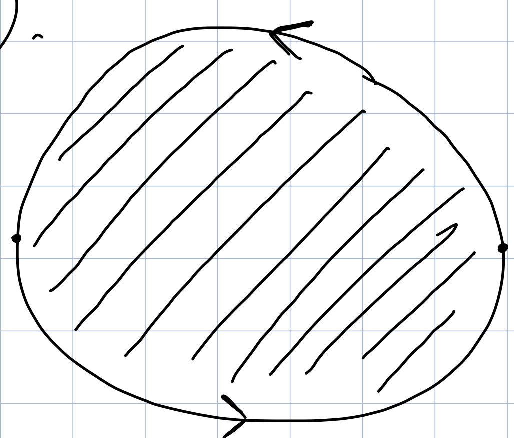


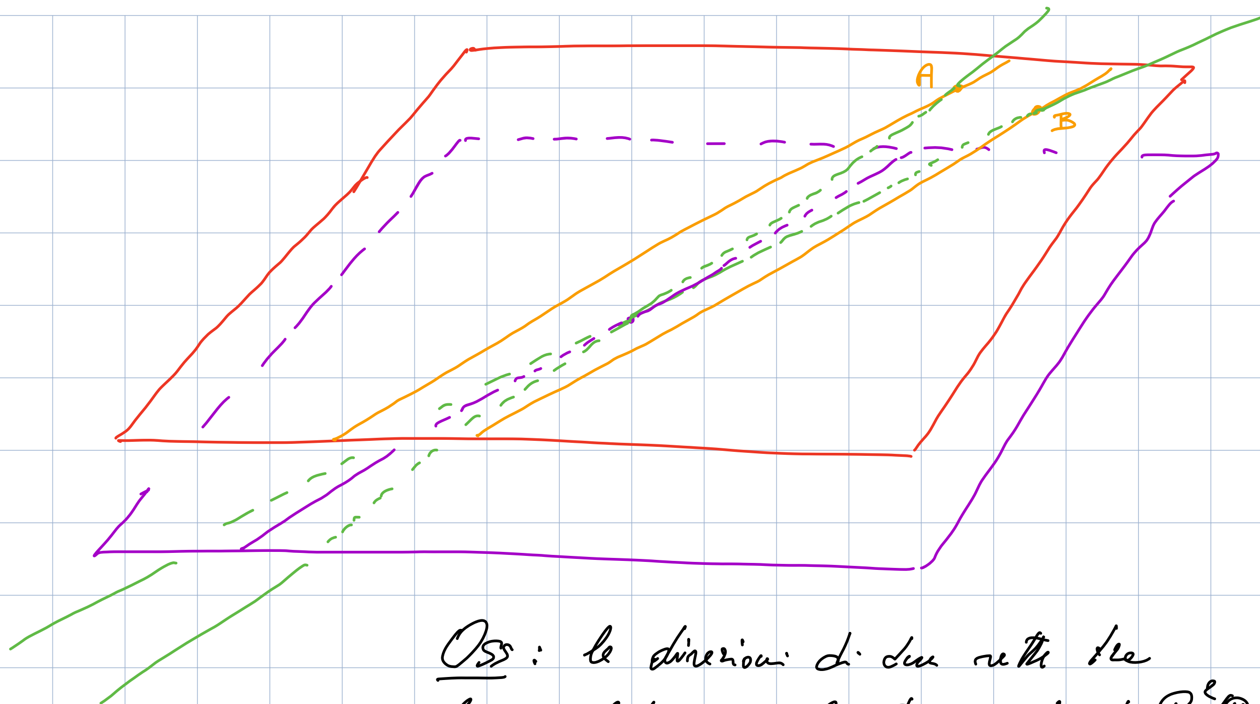
un pto di $\mathbb{R}^2$ vicino a cui $\rightarrow$ un pto di $\mathbb{P}^2(\mathbb{R})$ che non sta in $\mathbb{R}^2$

$$\mathbb{P}^2(\mathbb{R}) = \mathbb{R}^2 \cup \underbrace{\mathbb{P}^1(\mathbb{R})}_{\text{i punti all'}\infty\text{ di } \mathbb{R}^2}$$

i punti di $\mathbb{P}^2(\mathbb{R})$ che non sono in $\mathbb{R}^2$ sono le direzioni a cui si può tendere all'∞ in $\mathbb{R}^2$ (i due versi sono lo stesso).

$\Longrightarrow \mathbb{P}^2(\mathbb{R}) =$

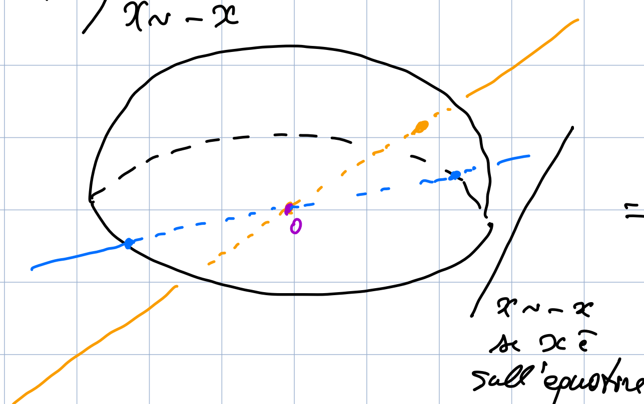




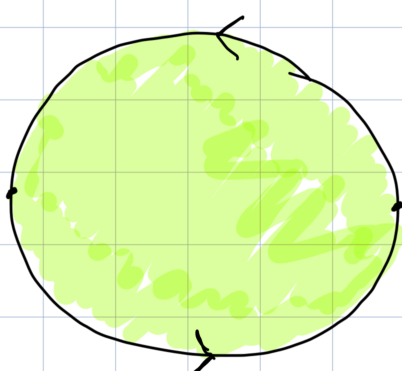
<u>Oss</u>: le direzioni di due rette tra loro parallele sono lo stesso punto di $\mathbb{P}^2(\mathbb{R})$.

——— o ———

$\mathbb{P}^2(\mathbb{R}) = S^2 \big/_{x \sim -x}$

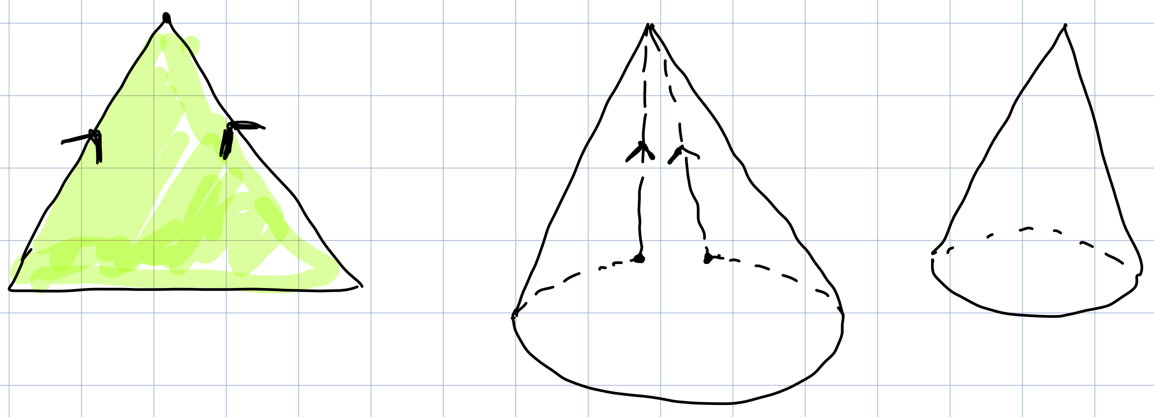
$=$ 

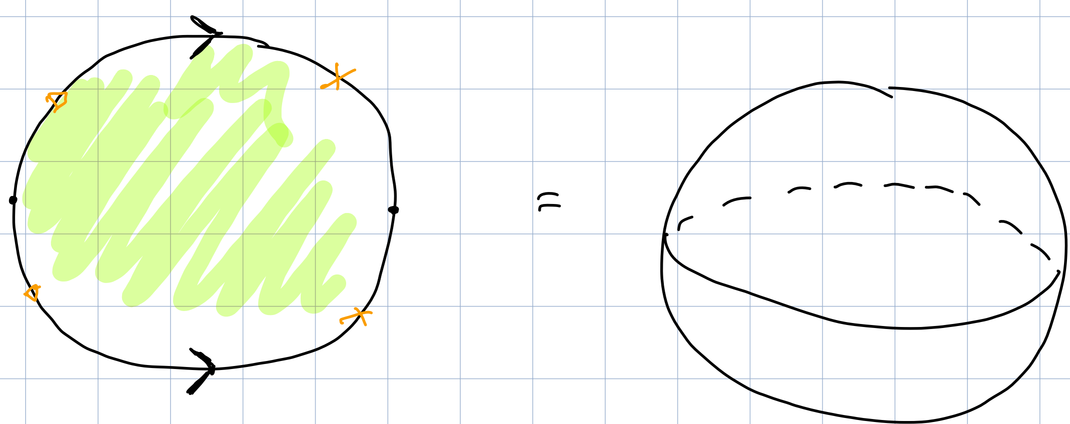
$\big/_{x \sim -x \text{ se } x \text{ è sull'equatore}}$ $= \{x \in S^2 : x_3 \geq 0\} \big/_{x \sim -x \text{ se } x_3 = 0}$

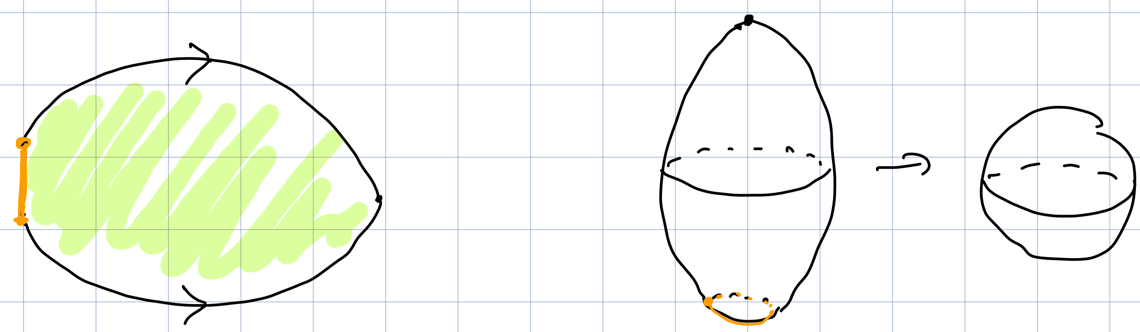
$=$ 

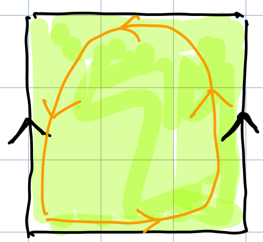
è un oggetto che non è possibile realizzare in $\mathbb{R}^2$.

Quali oggetti si possono realizzare incollando a coppie alcuni lati di un poligono?



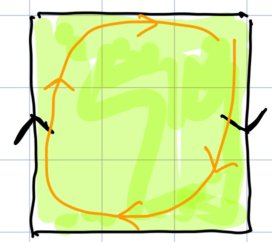






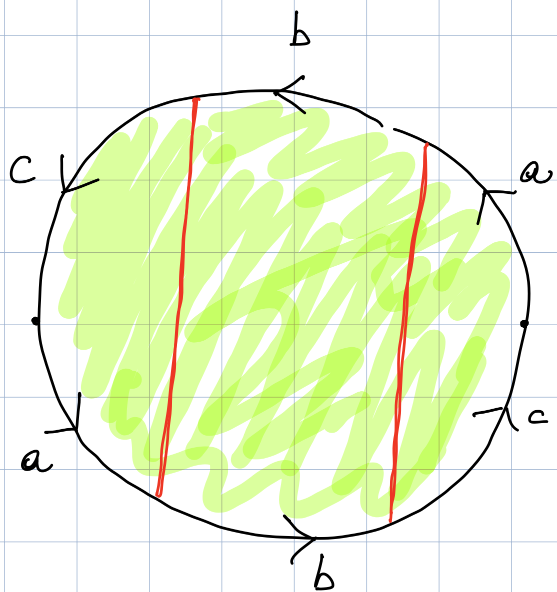
verso discorde

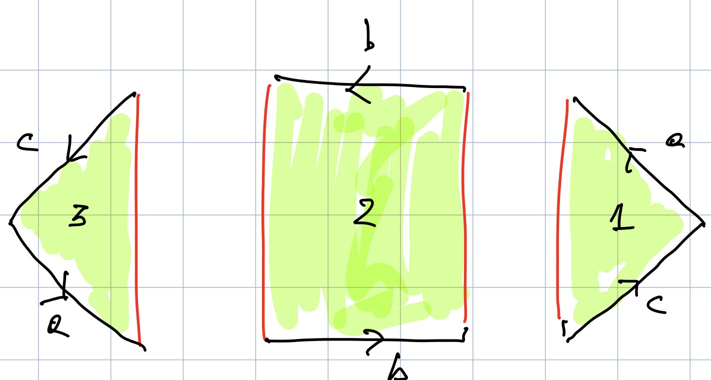
cilindro



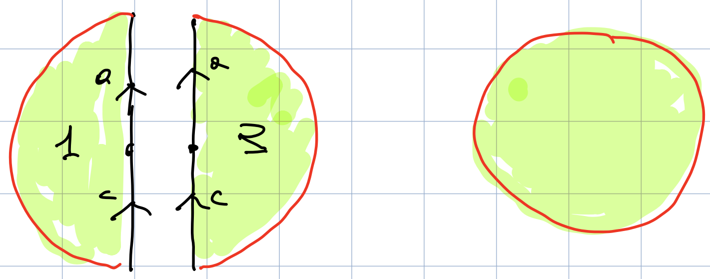
verso concorde

nastro di Möbius
(Moebius)



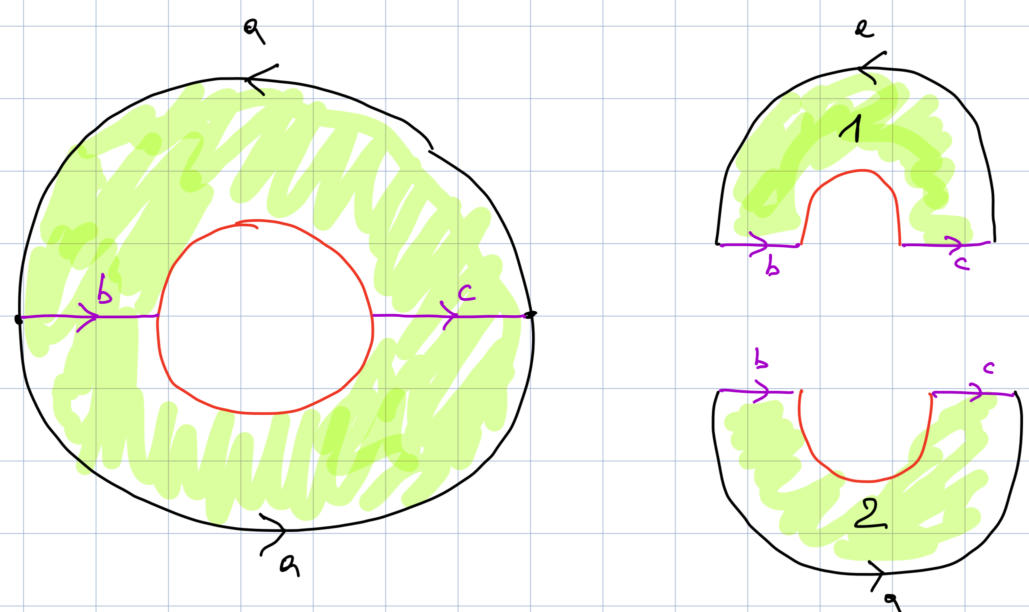


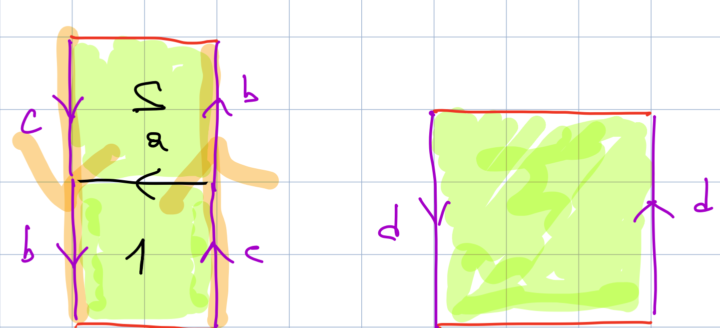
nastro di
Möbius



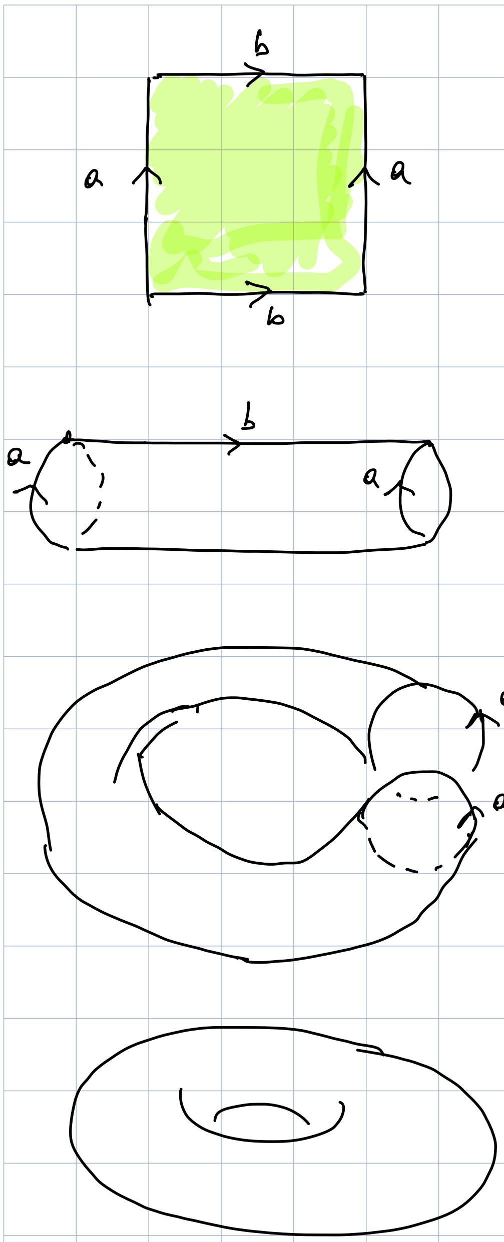
Piano proiettivo: incollare un disco al bordo del nastro di Möbius.

<u>Oss:</u> $\mathbb{P}^2(\mathbb{R}) \setminus$ disco $=$

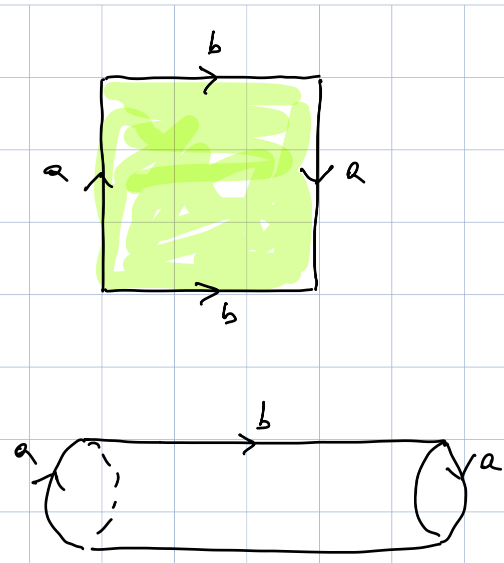




Möbius

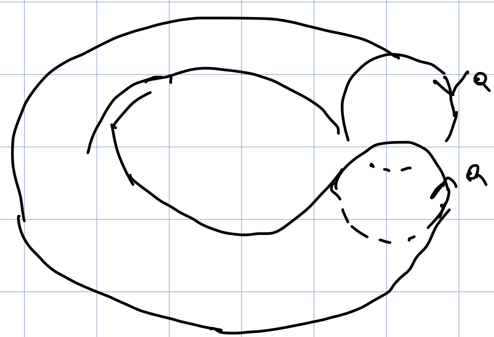


toro



bottiglia di Klein

fatto: non si realizza in $\mathbb{R}^3$



esercizio: cercare immagini.

<u>Esercizio</u>: determinare tutti gli oggetti che si possono ottenere da un quadrato identificando fra loro i lati a coppie (anche non opposti) –

Supp: sono tutti oggetti più visti; riconoscerli con tagli e incollamenti.

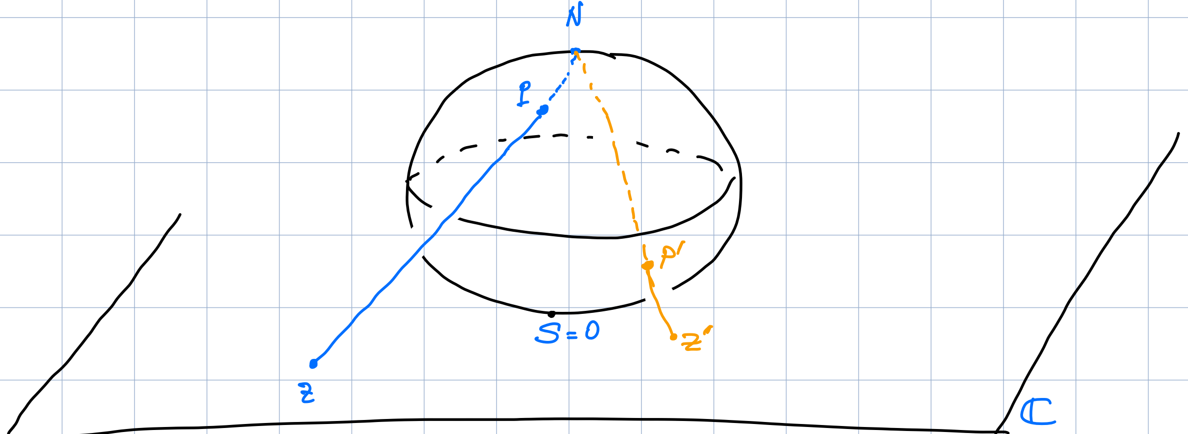
$$\mathbb{P}^1(\mathbb{C}) = \mathbb{C}^2 \setminus \{0\} \Big/ {\scriptstyle z \sim w \text{ se } w = \lambda \cdot z}$$

$$= \underset{\substack{\| \\ \mathbb{C} \times \{1\}}}{\mathbb{C}} \cup \underset{\substack{\| \\ \text{un punto}}}{\mathbb{P}^0(\mathbb{C})}$$

un punto $= \mathbb{C} \setminus \{0\} \Big/ {\scriptstyle u \sim v \text{ se } v = \lambda \cdot u}$

$\Rightarrow \mathbb{P}^1(\mathbb{C}) = \underset{\text{un piano reale}}{\mathbb{C}} \cup$ un pto all'∞ lo stesso in tutte le direz.

$= \text{sfera } \mathbb{S}^2.$



corrispondenza naturale tra $\mathbb{C}$ e $S^2 \setminus \{N\}$

i punti di $\mathbb{C}$ vicini a ∞ in qualsiasi direz.

$\longleftrightarrow$ punti di S^2 vicini a N

$$\Longrightarrow \quad \mathbb{P}^1(\mathbb{C}) = \mathbb{C} \cup \{\infty\} \longleftrightarrow S^2$$