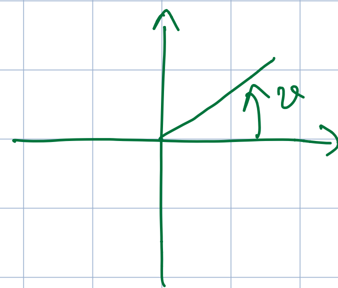
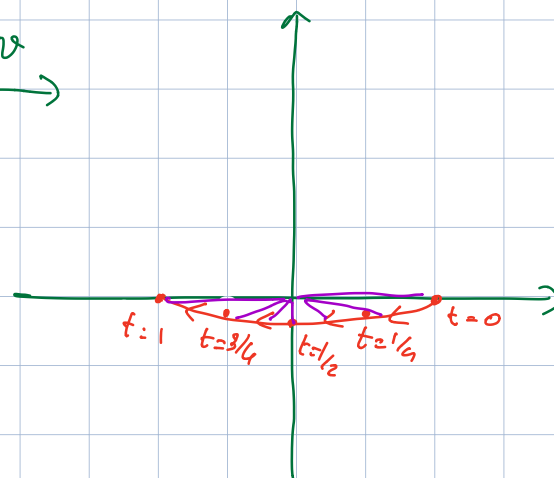
30/1/18 7

$$\int_\alpha \frac{-y\,dx + x\,dy}{x^2+y^2} \qquad \alpha : [0,1] \to \mathbb{R}^2 \quad \alpha(t) = \begin{pmatrix} 1-2t \\ t^2-t \end{pmatrix}$$

$$\int_\alpha d\vartheta = $$





$= \vartheta(1) - \vartheta(0)$

scegliendo ϑ coerente lungo α $= -\pi$

15/2/18 7

$$\int_{\partial Q} (\dots) \qquad Q = [0,1]\times[0,1]$$

$$\int_{\partial Q} \omega = \int_Q d\omega = \iint_Q \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy = \int_Q (-1-1)\,dx\,dy = -2$$

$\omega = f\,dx + g\,dy$

$$g = -(x + \sin(1-y^2)) \Rightarrow \frac{\partial g}{\partial x} = -1$$

8/6/21 4

$$\left\langle v \,\middle|\, \begin{pmatrix} 1-3i \\ 1+2i \end{pmatrix} \right\rangle_{\begin{pmatrix} 2 & 1+i \\ 1-i & 3 \end{pmatrix}} = 0 \quad ; \quad \langle z | w \rangle_A = w^* \cdot A \cdot z$$

$$(1+3i, 1-2i) \cdot \begin{pmatrix} 2 & 1+i \\ 1-i & 3 \end{pmatrix} \cdot \begin{pmatrix} z \\ w \end{pmatrix} = 0$$

$$\left(\cancel{2}+6i+1-2i-i-\cancel{2},\ 1+3i+i-\cancel{3}+\cancel{3}-6i\right)\cdot\begin{pmatrix} z \\ w \end{pmatrix}=0$$

$$(1+3i,\ 1-2i)\begin{pmatrix} z \\ w \end{pmatrix}=0$$

$$v=\alpha\cdot\begin{pmatrix} -1+2i \\ 1+3i \end{pmatrix} \qquad \alpha\in\mathbb{C}$$

(9) $\int_\alpha e^{4x}$ $\qquad \alpha(t)=\begin{pmatrix} \ln(t) \\ t^2-1 \end{pmatrix} \qquad \alpha:[1,2]\to\mathbb{R}^2$

$$\|\alpha'(t)\|=\sqrt{\left(\frac{1}{t}\right)^2+(2t)^2}=\sqrt{\frac{1}{t^2}+4t^2}=\frac{1}{t}\sqrt{1+4t^4}$$

$$\int_1^2 e^{4\ln(t)}\cdot\frac{1}{t}\sqrt{1+4t^4}\,dt=\int_1^2 t^3\sqrt{1+4t^4}\,dt$$

$$=\frac{2}{3}\cdot\frac{1}{16}\left(1+4t^4\right)^{3/2}\Big|_1^2=\frac{1}{24}\left(65^{3/2}-5^{3/2}\right)=\dots$$

16/2/21 (3) $\int_\alpha x-y$ $\qquad \alpha:[0,1]\to\mathbb{R}^2 \qquad \alpha(t)=\begin{pmatrix} t^2-t \\ t^2+t \end{pmatrix}$

$$\|\alpha'(t)\|=\sqrt{(2t-1)^2+(2t+1)^2}=\sqrt{8t^2+2}$$

$$\int_0^1(-2t)\cdot\sqrt{8t^2+2}\,dt=\frac{2}{3}\cdot\left(-\frac{1}{8}\right)\cdot\left(8t^2+2\right)^{3/2}\Big|_0^1=\dots$$

(10) $\int_\alpha xy(y\,dx + x\,dy)$ $\qquad \alpha : [-1,0] \to \mathbb{R}^2 \quad \alpha(t) = \begin{pmatrix} \sin(\pi/2 \cdot t) \\ \ln(2-t) \end{pmatrix}$

$$\omega = xy^2 dx + x^2 y\,dy = \frac{1}{2} d(x^2y^2) = d\left(\frac{1}{2}x^2y^2\right)$$

$$\int_\alpha \omega = \frac{1}{2}x^2y^2 \Big|_{\alpha(-1)}^{\alpha(0)} = \frac{1}{2}x^2y^2 \Big|_{(-1,\ \ln(3))}^{(0,\ \dots)} = -\frac{1}{2}\ln^2(3)$$

27/6/17-I (3) $\quad l \subset \mathbb{P}^2(\mathbb{R})$ per $[1:4:3]$ e $[-1:3:2]$

$r : \quad x_1 + x_2 - 3x_3 = 0$

$\pi : \mathbb{R}^3 \setminus \{0\} \to \mathbb{P}^2(\mathbb{R})$ $\qquad \pi^{-1}(l) \cup \{0\} = \mathrm{Span}\left(\begin{pmatrix}1\\4\\3\end{pmatrix}, \begin{pmatrix}-1\\3\\2\end{pmatrix}\right) = \tilde{l}$

$\pi^{-1}(r) : \quad x_1 + x_2 - 3x_3 = 0 = \tilde{r}$

$$l \cap r = \pi(\tilde{l} \cap \tilde{r} \setminus \{0\})$$

$\begin{pmatrix}1\\4\\3\end{pmatrix} + t\begin{pmatrix}-1\\3\\2\end{pmatrix}$ $\qquad (1-t) + (4+3t) - 3(3+2t) = 0$

$-4 - 4t = 0$

$t = -1$

$[2 : 1 : 1]$

10/1/17 ⑥ $[t-1 : -t : t+1]$ all'∞ di $8x^2 - y^2 + z^2 + 2xy - xz + yz + \dots = 0$

$$8(t-1)^2 - (-t)^2 + (t+1)^2 - (t-1)(t+1) + (-t)(t+1) = 0 \quad \dots$$

Libro arancio $\boxed{7}$ ⑥

$\mathbb{P}^2(\mathbb{R})$: $\{[3t-4 : -6 : 2t] : t \in \mathbb{R}\}$
$\{[t : t^2+2 : -2] : t \in \mathbb{R}\}$

$$\begin{pmatrix} 3t-4 \\ -6 \\ 2t \end{pmatrix} \sim \begin{pmatrix} s \\ s^2+2 \\ -2 \end{pmatrix}$$

tutti i det 2x2 di $\begin{pmatrix} \vdots & \vdots \\ \cdot & \cdot \end{pmatrix}$ devono essere 0 _

$$-12 - 2t(s^2+2) = 0 \qquad t = \frac{6}{s^2+2}$$

$$\left(\frac{\cancel{18}^{\,9}}{s^2+2} - \cancel{4}^{\,2}\right)\cdot\cancel{(-2)} = \frac{\cancel{12}^{\,3}}{s^2+2} \cdot s$$

$$-9 + 2(s^2+2) = 3s \qquad 2s^2 - 3s - 5 = 0$$

$$(2s-5)(s+1) = 0$$

$s = -1 \longrightarrow t = 2 \longrightarrow \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ ok

$s = 5/2 \longrightarrow t = \frac{24}{33} \longrightarrow \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}, \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$ no

$\boxed{7}$ (4) $z \in \mathbb{C}$ t.c. $\begin{pmatrix} 1+i & 2i \\ 2 & z \end{pmatrix}$ ha base ortonormale autovett.

A ha b.o.a. $\iff A^* A = A \cdot A^*$

$$\begin{pmatrix} 1-i & 2 \\ -2i & \bar{z} \end{pmatrix}\begin{pmatrix} 1+i & 2i \\ 2 & z \end{pmatrix} = \begin{pmatrix} 1+i & 2i \\ 2 & z \end{pmatrix}\begin{pmatrix} 1-i & 2 \\ -2i & \bar{z} \end{pmatrix}$$

$$\begin{cases} 2+4 = 2+4 \quad \checkmark \\ \cancel{2i} \cancel{+2} +2z = \cancel{2} \cancel{+2i} +2i\bar{z} \\ -2i+2+2\bar{z} = 2-2i-2iz \quad \checkmark \\ 4+|z|^2 = 4+|z|^2 \quad \checkmark \end{cases}$$

$z - i\bar{z} = 0$

$(\alpha + i\beta) - i(\alpha - i\beta) = 0$

$\alpha - \beta = 0$

$\beta - \alpha = 0$ $\qquad z = t(1+i) \quad t \in \mathbb{R}$

9/9/16 (6) $A \in M_{4\times 4}$ $B \subset A$ $B \in M_{2\times 2}$ $\det \neq 0$ due orlate hanno $\det = 0$. Quanto può valere il rango?

almeno 2.

2 sì
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3 sì
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

4 sì
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

29/1/19 ⑦

$$\overbrace{\left(2\cos(x-3y) + h(2x+y)\cdot\sin(x-3y)\right)}^{f}\,dx + \underbrace{\left(k\cos(x-3y) + 3(2x+y)\sin(x-3y)\right)}_{g}\,dy$$

esatta per $k, h = \ldots$?

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

$U \qquad \dfrac{\partial U}{\partial x} = f \qquad \dfrac{\partial U}{\partial y} = g$

$$U = (2x+y)\cdot\cos(x-3y)$$

$$\frac{\partial U}{\partial x} = 2\cdot\cos(x-3y) - (2x+y)\cdot\sin(x-3y) \qquad h = -1$$

$$\frac{\partial U}{\partial y} = \cos(x-3y) + 3(2x+y)\cdot\sin(x-3y) \qquad k = 1$$

29/1/16 - I ③

$\{[2t+1 : t+1 : t+4] : t \in \mathbb{R}\} \cap$ pti ass di $-2xy+y^2+z^2+\dots$

$-2(2t+1)\cdot(t+1)+(t+1)^2+(t+4)^2=0 \quad \dots$

⑤ tg. $x^3-3xy^2=2$ $\qquad x=2,\ y>0$

$8-6y^2=2 \qquad y=\pm 1 \qquad y=1$

$F = x^3-3xy^2-2 \qquad \text{grad}\, F_{(2,1)} = (6x^2-3y^2, -6xy)\Big|_{(2,1)}$

$= (18,-12) = 6(3,-2)$

$3x-2y=0 \qquad \text{Span}\begin{pmatrix}2\\3\end{pmatrix}$

Conti....

28/6/16 ③

$\{(x,y) : (y^2-4x^2+\sqrt{5})(2x+y+\sqrt{7})(x+2y-\sqrt{3})=0\}$

$[2:1]\ [2:-1] \qquad [1:-2]$

16/2/18 ④ trovare $\begin{pmatrix} i & 1+i \\ a & \sqrt{2}-1 \end{pmatrix} \cdot M$ diag con M unitaria per ... ?

$$\begin{pmatrix} i & 1+i \\ a & \sqrt{2}-1 \end{pmatrix} \cdot \begin{pmatrix} -i & \bar{a} \\ 1-i & \sqrt{2}-1 \end{pmatrix} = \begin{pmatrix} -i & \bar{a} \\ 1-i & \sqrt{2}-1 \end{pmatrix} \begin{pmatrix} i & 1+i \\ a & \sqrt{2}-1 \end{pmatrix}.$$

29/1/19 ③ $v \perp \begin{pmatrix} 3-i \\ 1+2i \end{pmatrix}$, $\|v\|=1$, $v_1 \in \mathbb{R}$.

$w = \begin{pmatrix} 1 \\ z \end{pmatrix}$; impongo $\left\langle w \,\middle|\, \begin{pmatrix} 3-i \\ 1+2i \end{pmatrix} \right\rangle = 0$ e poi $v = \pm \frac{1}{\|w\|} \cdot w$

$$1 \cdot (3+i) + z(1-2i) = 0$$

$$z = -\frac{3+i}{1-2i} = -\frac{(3+i)(1+2i)}{5} = \dots$$