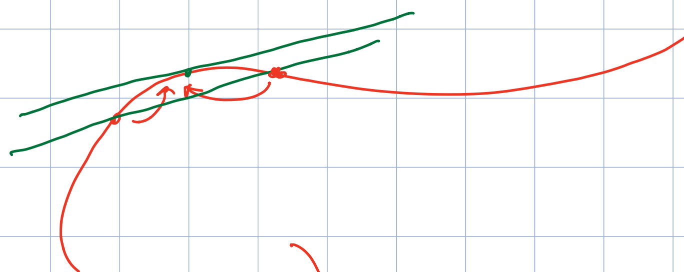
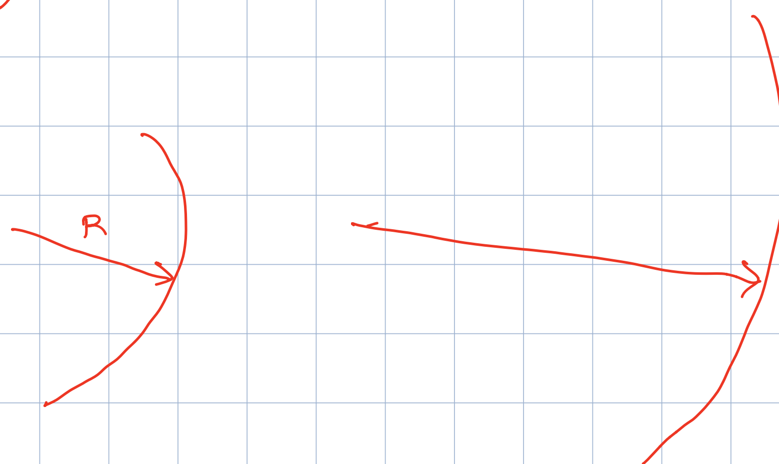
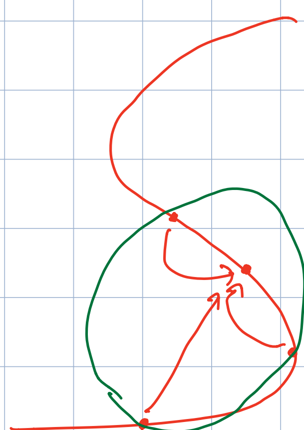
Geom - Civ - 28/5/21

Q : curvatura curve piane



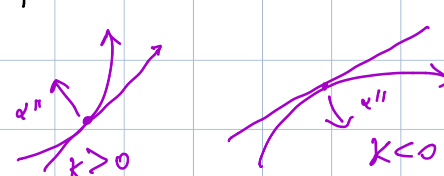


$K = \frac{1}{R}$



α in p.d'a circonf osc. esiste se $\alpha''(t_0) \neq 0$ e ha raggio $1/\|\alpha''(t_0)\|$; $K = \|\alpha''(t_0)\|$

• segno per curve orientate :



- β qualsiasi orientata

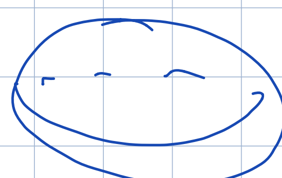
$$\kappa(t) = \frac{\det(\beta'(t) \ \ \beta''(t))}{\|\beta'(t)\|^3}$$

Q: modelli affini quadriche

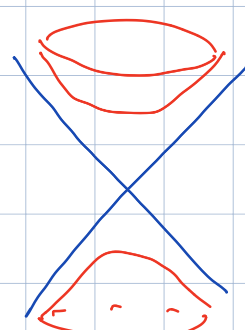
$\mathcal{Q} = \left\{ x \in \mathbb{R}^3 : {}^t\begin{pmatrix} x \\ 1 \end{pmatrix} \cdot A \cdot \begin{pmatrix} x \\ 1 \end{pmatrix} = 0 \right\}$ A simm. $\det(A) \neq 0$

$\exists$ sempre una trasf. affine di $\mathbb{R}^3$ che trasforma $\mathcal{Q}$ in uno dei 6 modelli affini:

- $x^2+y^2+z^2+1=0$ $\emptyset$
- $x^2+y^2+z^2=1$ ellissoide

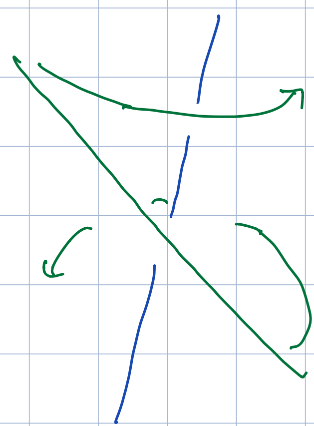


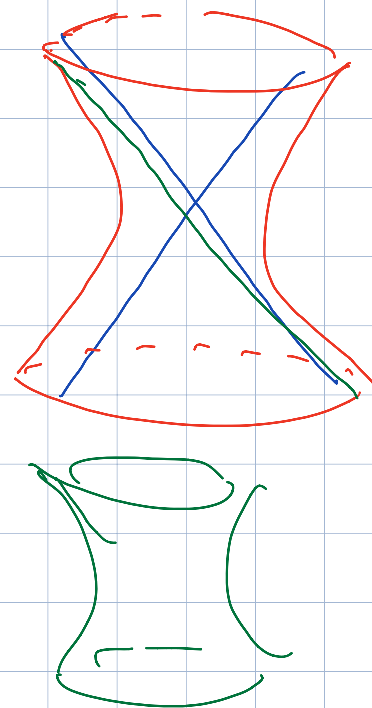
- $z = x^2+y^2$ parab. ell.
- $x^2+y^2+1 = z^2$
  iperb. a 2 falde / ell.



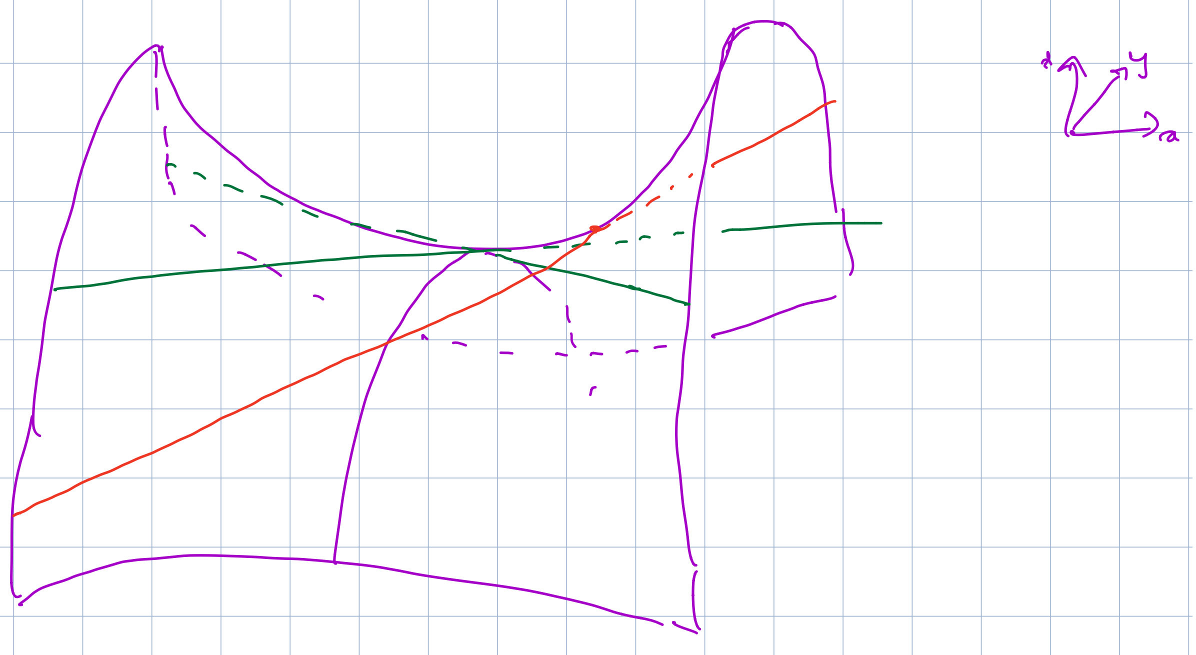
- $x^2 + y^2 = 1 + z^2$

iperb. 1 falda / iperb.





- $z = x^2 - y^2$ parab. iperb / sella.



• scrivere matrice di ognuna delle equaz.

• dire a quale modello ci si trasforma in base ai segni degli autoval. di Q e A

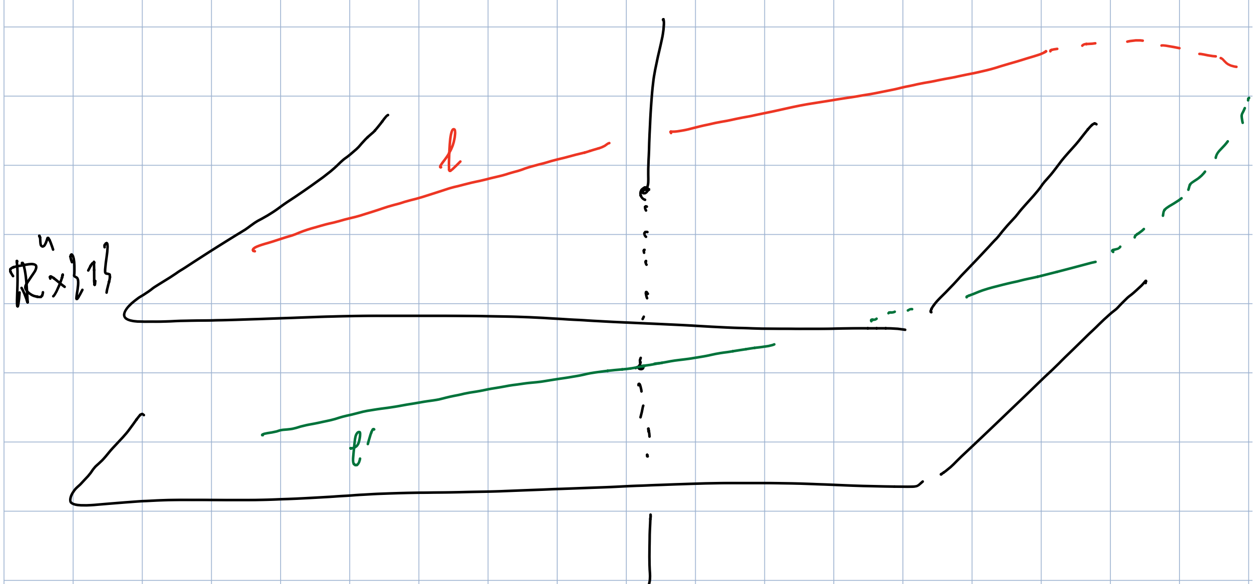
Q: punto all'∞.

$$\mathbb{P}^n(\mathbb{R}) = \left(\mathbb{R}^{n+1}\setminus\{0\}\right)\Big/_{\substack{x\sim y \\ \text{se } y=\lambda\cdot x}} = \mathbb{R}^n \cup \overbrace{\mathbb{P}^{n-1}(\mathbb{R})}^{\text{pti all'}\infty\text{ di }\mathbb{R}^n}$$

$$\mathbb{R}^n = \mathbb{R}^n\times\{1\} \subset \mathbb{R}^{n+1}, \qquad \mathbb{P}^{n-1}(\mathbb{R}) = \mathbb{P}(\mathbb{R}^n\times\{0\})$$

$l \subset \mathbb{R}^n$ retta affine ha come pto all'∞

$[l'\setminus\{0\}] \subset \mathbb{P}^{n-1}(\mathbb{R})$ $\qquad l' \subset \mathbb{R}^n\times\{0\}$ retta $/\!/\ l$



$\mathcal{L}$ qualsiasi $\longrightarrow \mathcal{L}_\infty$ "le direzioni delle rette affini lungo cui $\mathcal{L}$ tende all'∞"

$$\overline{\mathcal{L}} = \mathcal{L} \cup \mathcal{L}_\infty$$

$\mathcal{L}$ def. da equaz. poli di grado d in n var

$\overline{\mathcal{L}}$ def. dalle* stesse equaz. con 1 var. in più u e rese omogenee di grado d

$$\mathcal{L}_\infty = \{ [x] \in \overline{\mathcal{L}} : u = 0 \}$$

Q: modelli metrici coniche

ellisse: P_1, P_2 $\quad d(P_1, P_2) = 2k \quad h > k \geq 0$

$$\mathcal{E} = \{ A \in \mathbb{R}^2 : d(A, P_1) + d(A, P_2) = 2h \}$$

$P_1 = (-k, 0) \qquad P_2 = (k, 0) \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

iperbole: $P_1, P_2 \quad d(P_1, P_2) = 2k \quad 0 < h < k$

$$\mathcal{I} = \{ A : | d(A, P_1) - d(A, P_2) | = 2h \}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

parabola : $P \notin \ell$

$$\mathcal{P} = \{ A : d(A, P) = d(A, \ell) \}$$

A meno di isom. $\ell : y = h$ $P = (0, -h)$

$$y = a \cdot x^2$$

Q : prod. scal. herm.

V sp. vett. su $\mathbb{C}$ $f : V \times V \to \mathbb{C}$

- sesquilineare : lin. a sx , antilin. a dx
- hermitiano : $f(u,v) = \overline{f(v,u)}$ $(\Rightarrow f(u,u) \in \mathbb{R})$
- def. pos : $f(u,u) > 0 \quad \forall u \neq 0$.

$$\langle z | w \rangle_{\mathbb{C}^n} = w^* \cdot z$$

$f \rightsquigarrow \|\cdot\|$ $\|v\| = \sqrt{f(v,v)}$ $d(u,v) = \|u - v\|$

Q' : Come sono fatti i prod. scal. herm. su $\mathbb{C}^n$?

Data $A \in M_{n\times n}(\mathbb{C})$ pongo

$$\langle \cdot | \cdot \rangle_A : \mathbb{C}^n \times \mathbb{C}^n \longrightarrow \mathbb{C}$$
$$\langle u, v \rangle \longmapsto v^* \cdot A \cdot u .$$

- $\langle \cdot | \cdot \rangle_A$ è sempre sesquil. e ogni sesquil. è siffatta
- $\langle \cdot | \cdot \rangle_A$ hermitiana $\Longleftrightarrow$ $A^* = A$ (A hermitiana)
- $\langle \cdot | \cdot \rangle_A$ def. pos (A hermitiana) $\Longleftrightarrow$ tutti gli autoval. di A sono >0 $\Longleftrightarrow$ $d_1, \dots, d_n > 0$

Q: Gauss - Green.

$\Omega \subset \mathbb{R}^2$ aperto limitato
$\partial\Omega$ unione finita di curve regolari
ω 1-forma definita su un aperto che contiene $\overline{\Omega} = \Omega \cup \partial\Omega$

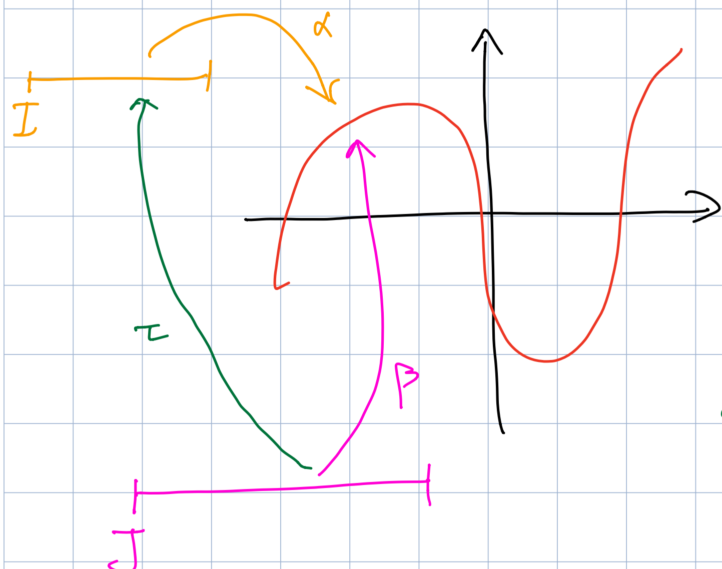
$$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega$$

$\partial\Omega$ orientato così da lasciare Ω a sinistra
$d\omega = d(f\,dx + g\,dy) = \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy$

Q: cambio di parametro

$\alpha : I \to \mathbb{R}^n$ $\qquad I \subset \mathbb{R}$ intervallo

Diciamo che $\beta : J \to \mathbb{R}^n$ è ottenuta da α per cambio di parametro se $\exists\, \tau : J \to I$ C^∞ e monotona t.c.



$\beta = \alpha \circ \tau$

$\beta(s) = \alpha(\tau(s))$

$\sigma = \tau^{-1}$

$\alpha(t) = \beta(\sigma(t))$

Q: come ottenere il p.d'a.?

β è in p.d'a. se $\|\beta'(s)\| = 1 \quad \forall s.$

$\alpha : [a,b] \to \mathbb{R}^n \qquad \sigma(t) = \int_a^t \|\alpha'(u)\|\,du$

$\sigma : [a,b] \to [0,L] \qquad (\alpha'(t) \neq 0 \quad \forall t)$

$\tau : [0,L] \to [a,b] \qquad \beta(s) = \alpha(\tau(s))$

Q: teorema spettrale

$A \in M_{n\times n}(\mathbb{R})$ $\quad$ A simmetrica se ${}^tA = A$

$M \in M_{n\times n}(\mathbb{R})$ $\quad$ M ortogonale se ${}^tM = M^{-1}$

cioè le colonne di M sono base ortonormale

Teo: A è simmetrica $\Longleftrightarrow$ $\exists M$ ortogonale che la diagonalizza, cioè

$$M^{-1} \cdot A \cdot M = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$