Geom - Lin - 26/5/21

21/7/20 8 $\quad 12x^3 - 5x^2y - 17xy^2 + 10y^3 = 0$

$(x-y)(12x^2 + 7xy - 10y^2)$

$\dfrac{-7 \pm \sqrt{49+480}}{24} = \dfrac{-7\pm 23}{24} \begin{cases} \frac{2}{3} \\ -\frac{5}{4} \end{cases}$

$\begin{array}{r} 23 \\ 23 \\ \hline 69 \\ 46 \\ \hline \end{array}$

$(x-y)(3x-2y)(4x+5y) = 0$

$[1:1]\ [2:3]\ [5:-4]$

9 $\quad f(x,y) = \sin(2x-3y) + e^{5x+7y} - 1$

$\mathcal{C} = \{(x,y) : f(x,y) = 0\}$

$f(0,0) = \sin(0) + e^0 - 1 = 0$ ✓

$\mathcal{C}$ curva di cento se $\text{grad}(f)(0,0) \neq 0$ e in tal caso la tg è $\perp$ grad.

$\dfrac{\partial f}{\partial x}(0,0) = 2\cdot\cos(0) + 5\cdot e^0 = 7$

$\dfrac{\partial f}{\partial y}(0,0) = -3\cdot\cos(0) + 7e^0 = 4$

Sì curva ; $tg = \text{Span}\begin{pmatrix} -4 \\ 7 \end{pmatrix}$

⑩ $\displaystyle\int_{\partial D^2} \omega = \int_{D^2} d\omega$ $\qquad \omega = \underbrace{\left(f(x)+y\right)}_{A} dx + \underbrace{\left(g(y)-x\right)}_{B} dy$

$$d\omega = \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right) dx\,dy = -2\,dx\,dy$$

-2π

25/6/19 ④

$A \in \mathcal{M}_{n\times n}(\mathbb{R})$, ${}^tA = -A \implies \exists M \in \mathcal{M}_{n\times n}(\mathbb{R})$, ${}^tM = M^{-1}$

t.c. $M^{-1}\cdot A\cdot M = \begin{pmatrix} \boxed{\begin{matrix} 0 & \alpha_1 \\ -\alpha_1 & 0 \end{matrix}} & & & O \\ & \boxed{\begin{matrix} 0 & \alpha_2 \\ -\alpha_2 & 0 \end{matrix}} & & \\ & & \ddots & \\ O & & & \boxed{k} & & \\ & & & & 0 & \\ & & & & & \ddots & \\ & & & & & & 0 \end{pmatrix}$

dove gli autoval. di A sono $\pm i\alpha_1, \pm i\alpha_2, \dots, \pm i\alpha_k, 0, \dots, 0$

3×3 : $\begin{pmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ se A ha autoval $\pm i\alpha, 0$

$$\begin{pmatrix} 0 & -3 & 7 \\ 3 & 0 & -5 \\ -7 & 5 & 0 \end{pmatrix}$$

$$P_A(t) = \det \begin{pmatrix} t & 3 & -7 \\ -3 & t & 5 \\ 7 & -5 & t \end{pmatrix}$$

$= t^3 + 3\cdot5\cdot7 - 3\cdot5\cdot7 + 49t + 9t + t$

$= t^3 + 59t = t\,(t^2+59)$ $\qquad 0, \pm i\sqrt{59}$ $\qquad \alpha = \pm\sqrt{59}$

16/7/19 (1) $X : x_1 + x_2 + x_3 = 0$

$$A = \begin{pmatrix} -9 & 19 & 1 \\ 0 & 17 & 6 \\ 16 & -29 & 0 \end{pmatrix}$$

$f : X \to X$, $f(x) = A \cdot x$, Autoval e base diago.

$g : V \to W$ lineare $U \subset V$, $Z \subset W$ sottosp. vett.
se ho $g(U) \subset Z$ allora $f : U \to Z$ data $f(u) = g(u)$ è lin.

Per noi: $V = W = \mathbb{R}^2$, $g(x) = A \cdot x$, $U = Z = X$

Dovrei verificare che $g(X) \subset X$: cioè che se x soddisfa l'equaz. di X lo fa anche $g(x)$, infatti:

$$(-9x_1 + 19x_2 + x_3) + (17x_2 + 6x_3) + (16x_1 - 29x_2)$$
$$= 7x_1 + 7x_2 + 7x_3 = 7(x_1 + x_2 + x_3) = 7 \cdot 0 = 0 \quad \underline{\underline{ok}}$$

Per trovare gli autoval. di f serve $[f]_{\mathcal{B}}^{\mathcal{B}}$:

$$\mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right)$$

$$f\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -9 & 19 & 1 \\ 0 & 17 & 6 \\ 16 & -29 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -10 \\ -6 \\ 16 \end{pmatrix} = -10 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - 6 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$f\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 18 \\ 11 \\ -29 \end{pmatrix} = 18 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + 11 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$[f]_{\mathcal{B}}^{\mathcal{B}} = \begin{pmatrix} -10 & 18 \\ -6 & 11 \end{pmatrix} \qquad tr = 1 \qquad det = -110 + 108 = -2$$

$\lambda_1 = 2 \qquad \lambda_2 = -1$

Per trovare gli autovett. di f trovo quelli di $[f]^{\mathcal{B}}_{\mathcal{B}}$ e poi li trasformo in vett. di X.

$v_{1,2}$ di $[f]^{\mathcal{B}}_{\mathcal{B}}$

$$v_1: \quad \begin{pmatrix} -10 & 18 \\ -6 & 11 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 2 \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{cases} -10x + 18y = 2x \\ -6x + 11y = 2y \end{cases}$$

$$\begin{cases} \cancel{-12x + 18y = 0} \\ -6x + 9y = 0 \end{cases} \qquad v_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$v_2: \quad -10x + 18y = -x \qquad -9x + 18y = 0 \qquad v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$u_{1,2}$ di f:

$$u_1 = 3 \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + 2 \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}$$

$$u_2 = 2 \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

(5) determinare tipo proiettivo quadrica ...

$$\mathcal{Q} = \{ [x] \in \mathbb{P}^3(\mathbb{R}) \ : \ {}^t x \cdot A \cdot x = 0 \} \qquad A \in \mathcal{M}_{4\times 4}(\mathbb{R})$$

$${}^t A = A, \quad \det(A) \neq 0$$

Segni autoval. di A: 4 concordi, 3 concordi, 2 concordi

$+ + + + \qquad + + + - \qquad + + - -$

$\emptyset$ ellissoide proiett. iperb. proiett.

$$-y^2+w^2+2xy-4xz+6yz+2yw=0$$

$$A=\begin{pmatrix} 0 & 1 & -2 & 0 \\ 1 & -1 & 3 & 1 \\ -2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$\boxed{d_2<0}$ $\quad d_3=-6-6+4<0$

$$d_4=\begin{pmatrix} 0 & 1 & -2 & 0 \\ 1 & -2 & 3 & 0 \\ -2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}=-6-6+8<0$$

sgn autoval: $\boxed{+\ -}\ +\ +$: ellissoide proiett.

$\quad - \ \ +$

15/9/20 (6) $\quad A_t=\begin{pmatrix} t-1 & t-2 \\ 2 & 1-t \end{pmatrix}$

Trovare t per cui

$A_t=\alpha\cdot M$ $\quad M$ ortog e dire se M è rotaz o rifl.

$A=(u\ v)$ ortog. se $\|u\|=\|v\|=1$ e $\langle u|v\rangle=0$

$A=(u,v)=\alpha\cdot$ ortog. se $\|u\|=\|v\|$ e $\langle u|v\rangle=0$

$\langle u|v\rangle=0$: $\quad (t-1)(t-2)+2(1-t)=0 \quad (t-1)(t-2-2)=0$

$t=1,\quad t=4$

$t=1$ $\quad \|u\| \neq \|v\|$ $\quad$ no

$t=4$ $\quad \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix}$ $\quad$ ok

$\det = -13 \implies \sqrt{13} \cdot$ rifl.

⑦ Tipo $\quad 3x^2+4xy+5y^2-2x+2y+1=0$.

$$\begin{pmatrix} 3 & 2 & -1 \\ 2 & 5 & +1 \\ -1 & +1 & 1 \end{pmatrix}$$

$d_1>0 \quad d_2>0$

$d_3 = 15-2-2-5-4-3 < 0$

$+ + -$

ellisse

⑧ Tipo $\quad x^2-z^2+4xy-4yz+4y+4z-3=0$

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 2 & 2 & -3 \end{pmatrix}$$

$d_2<0$

$d_3 = 4-4=0$

$d_4 \neq 0 \implies$ parab iperb.

$$d_4 = \begin{vmatrix} 1 & 2 & 0 & 0 \\ 0 & -4 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 2 & 2 & -3 \end{vmatrix} = \begin{vmatrix} 0 & 2 & -4 \\ 0 & 1 & -1 \\ 2 & 2 & -3 \end{vmatrix} \neq 0$$

ok

(9) $\alpha(t) = \begin{pmatrix} t^3 + \frac{3}{5}t + 1 \\ 5t^2 + 4t + 3 \end{pmatrix}$ $\kappa(0)$, $\operatorname{sgn}(\kappa(t))$

$\alpha'(t) = \begin{pmatrix} 3t^2 + 3/5 \\ 10t + 4 \end{pmatrix}$ $\alpha''(t) = \begin{pmatrix} 6t \\ 10 \end{pmatrix}$

$\alpha'(0) = \begin{pmatrix} 3/5 \\ 4 \end{pmatrix}$ $\alpha''(0) = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$

$$\kappa(0) = \frac{\det\begin{pmatrix} 3/5 & 0 \\ 4 & 10 \end{pmatrix}}{\left((3/5)^2 + 4^2\right)^{3/2}} = \cdots$$

$$\operatorname{sgn}(\kappa(t)) = \operatorname{sgn} \det\begin{pmatrix} 3t^2 + 3/5 & 6t \\ 10t + 4 & 10 \end{pmatrix}$$

$$= \operatorname{sgn}\left(30t^2 + 6 - 60t^2 - 24t\right)$$

$$= -\operatorname{sgn}\left(5t^2 + 4t - 1\right) = -\operatorname{sgn}(5t - 1)(t + 1)$$

neg. per $t < -1$, $t > 1/5$ ......

(10) $\boxed{\displaystyle\int_\alpha e^{x+y^2}\left(dx+2y\,dy\right)}$ $\qquad \alpha(t) = \begin{pmatrix} \ln(1+t) \\ 1-t \end{pmatrix}$

$t \in [0,1]$

$$\int_0^1 e^{\log(1+t)+(1-t)^2}\left(\frac{1}{1+t}+2(1-t)(-1)\right)dt$$

$\omega = f\,dx + g\,dy$ chiusa?

$f = e^{x+y^2}$ $\qquad g = 2y\,e^{x+y^2}$

$\dfrac{\partial f}{\partial y} = 2y e^{x+y^2}$ $\qquad \dfrac{\partial g}{\partial x} = 2y e^{x+y^2}$ $\qquad$ sì

$$U(x,y) = e^{x+y^2} \Longrightarrow \int_\alpha U = U(\alpha(1)) - U(\alpha(0))$$

$$= U\begin{pmatrix} \ln(2) \\ 0 \end{pmatrix} - U\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= e^{\ln(2)} - e^1 = 2 - e$$

8/1/21 (1) $\begin{pmatrix} 2k^2+7k+1 & -k^2-3k \\ 2k^2+14k+24 & -k^2-6k-11 \end{pmatrix}$

$tr = k^2+k-10$

$$\det = \begin{vmatrix} 2k^2+7k+1 & -k^2-3k \\ 7k+23 & -3k-11 \end{vmatrix}$$

$$= \begin{vmatrix} k+1 & -k^2-3k \\ k+1 & -3k-11 \end{vmatrix}$$

$$= (k+1) \begin{vmatrix} 1 & -k^2-3k \\ 1 & -3k-11 \end{vmatrix} = (k+1)(-3k-11+k^2+3k)$$
$$= (k+1)(k^2-11)$$

$\lambda_1 + \lambda_2 = k^2+k-10$

$\lambda_1 \cdot \lambda_2 = (k+1)(k^2-11)$

$\lambda_1 = k+1, \ \lambda_2 = k^2-11$

$\lambda_1 = \lambda_2$ per $\quad k^2-11 = k+1 \quad k^2-k-12 = 0$

$(k-4)(k+3) = 0$

Diag. per $k \neq -3, 4$

$k = -3$: $\begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix}$ sì

$k = 4$: $\begin{pmatrix} * & -16-12 \\ & * \end{pmatrix}$ no

② $P = \text{Span}\left(\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right)$

punto di P più vicino a $\begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix}$.

$$\pi_P \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix} - \pi_{P^\perp} \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix}$$

$$P^\perp = \text{Span}\left(\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right) = \text{Span}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$2+3-5$ ✓ $4+1-5$ ✓

$$= \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \middle| \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix} \right\rangle}{\left\| \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \right\|^2} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix} - \frac{14+6+10}{4+1+25} \cdot \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ -3 \end{pmatrix}$$

③ $\langle \cdot | \cdot \rangle_A$ $\quad A = \begin{pmatrix} 20 & k^2+3 \\ 5k-1 & k \end{pmatrix}$ prod. scal.?

simm: $k^2+3 = 5k-1$ $\quad k^2-5k+4=0$

$(k-1)(k-4)=0$

def. pos.

$k=1$: $\begin{pmatrix} 20 & 4 \\ 4 & 1 \end{pmatrix}$ $\quad d_1 = 20 > 0$, $\quad d_2 = 20-16 > 0$ Sì

$k=4$: $\begin{pmatrix} 20 & 19 \\ 19 & 4 \end{pmatrix}$ $\quad d_1 = 20 > 0$, $\quad d_2 = 80 - 19^2 < 0$ No

④ $\begin{pmatrix} 1 & 2 \\ 2 & -3 \end{pmatrix}$

t^2+2t-7 $\quad \lambda_{1,2} = -1 \pm \sqrt{1+7} = -1 \pm 2\sqrt{2}$

$v_{1,2}$: $x+2y = (-1\pm 2\sqrt{2})x$ $\quad 2y = (-2 \pm 2\sqrt{2})x$

...

$-y = (1 \mp \sqrt{2})x$

$v_1 = \begin{pmatrix} -1 \\ 1-\sqrt{2} \end{pmatrix}$ $\quad v_2 = \begin{pmatrix} -1 \\ 1+\sqrt{2} \end{pmatrix}$

Già garantito $v_1 \perp v_2$: infatti

$$1 + 1 - 2 = 0$$