Geom - Civ - 20/5/21

15.1.1 $\int_\alpha \omega$

(b) $\omega(x,y) = x\,dy - y\,dx \qquad \alpha : [0,v] \to \mathbb{R}^2 \qquad \alpha(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

$$\int_\alpha * \;\to\; \int_\alpha f \longrightarrow \int_a^b f(\alpha(t)) \cdot \|\alpha'(t)\|\,dt$$

$$\int_\alpha * \;\to\; \int_\alpha (f\,dx + g\,dy) \longrightarrow \int_a^b \Big(f(\alpha(t)) \cdot X'(t) + g(\alpha(t)) \cdot Y'(t) \Big) dt$$

$$\int_0^v \Big(\cos(t) \cdot \cos(t) - \sin(t) \cdot (-\sin(t)) \Big) dt = \int_0^v dt = v$$

(d) $\omega(x,y,z) = 2y^2 z\,dx - 3xz^2\,dy \qquad \alpha : [0,1] \to \mathbb{R}^3$

$$\alpha(t) = \begin{pmatrix} t - e^t \\ 1 - e^{2t} \\ t^2 + 3e^t \end{pmatrix}$$

1-forme su $\Omega \subset \mathbb{R}^3$

- $\int_\alpha \omega = \int_\alpha f\,dx + g\,dy + h\,dz$
  $= \int_a^b \Big(f(\alpha(t)) \cdot X'(t) + g(\alpha(t)) \cdot Y'(t) + h(\alpha(t)) \cdot Z'(t) \Big) dt$

- $dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz$

- $\int_\alpha dU = U(\alpha(b)) - U(\alpha(a))$

Come decidere se α è $\alpha = dU$: altra storia

$$\omega(x,y,z) = 2y^2 z\, dx - 3xz^2 dy \qquad \alpha : [0,1] \to \mathbb{R}^3$$

$$\alpha(t) = \begin{pmatrix} t - e^t \\ 1 - e^{2t} \\ t^2 + 3e^t \end{pmatrix}$$

$$\int_\alpha \omega = \int_0^1 \Big(2(1-e^{2t})\cdot(t^2+3e^t)(1-e^t) - 3(t-e^t)(t^2+3e^t)^2(-2e^{2t}) \Big) dt$$

$$= \dots$$

$$\int t^n \cdot e^{kt} = t^n \cdot \frac{1}{k} e^{kt} - \int n t^{n-1} \frac{1}{k} e^{kt}$$

15.2.1 calcolare df

(b) $f(x,y) = \tan\left(\dfrac{1 - x^2 y^3}{y - 2\sin(3x-y)}\right)$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy =$$

$$= \left(1 + \tan(\ )^2\right) \cdot \frac{-2xy^3\,(y - 2\sin(3x-y)) + 2\cos(3x-y)\cdot 3}{(y - 2\sin(3x-y))^2}\, dx$$

$$+ \left(1 + \tan(\ )^2\right) \cdot \frac{-3x^2y^2(\dots) - (1 - 2\cos(3x-y)\cdot(-1))}{(\quad \cdot - \quad)^2}\, dy$$

15.2.2 Per quali $f : \mathbb{R}^2 \to \mathbb{R}$ le $\omega(x,y) = f(x,y)\, dy$ è chiusa?

$$\omega = 0 \cdot dx + f \cdot dy$$

chiusa $\Leftrightarrow \dfrac{\partial 0}{\partial y} = \dfrac{\partial f}{\partial x} \Leftrightarrow \dfrac{\partial f}{\partial x} = 0$

$\Leftrightarrow f(x,y) = f(y)$

15.2.3 Per quali k la ω ha pot?

(b) $\omega(x,y) = \cos(3x^{k(k+1)} - 2y^{k(k+2)})\,(x\,dx - y^2 dy)$

ha pot $\Leftrightarrow$ esatta $\Leftrightarrow$ chiusa (in $\mathbb{R}^2$)

$$\Leftrightarrow \frac{\partial}{\partial y}\left(x \cdot \cos(3x^{k(k+1)} - 2y^{k(k+2)})\right)$$

$$= \frac{\partial}{\partial x}\left(-y^2 \cdot \cos(3x^{k(k+1)} - 2y^{k(k+2)})\right)$$

$$\Leftrightarrow -\sin(\ldots) \cdot x \cdot k(k+2) \cdot (-2y^{k(k+2)-1})$$

$$= -\sin(\ldots) \cdot -y^2 \cdot k(k+1) \cdot (3x^{k(k+1)-1})$$

$$\begin{cases} 2k(k+2) = 3k(k+1) \checkmark \\ k(k+1) - 1 = 1 \checkmark \\ k(k+2) - 1 = 2 \checkmark \end{cases} \qquad \begin{cases} k(k+1) = 2 \\ k(k+2) = 3 \end{cases} \Rightarrow \boxed{k = 1}$$

Oppure

$\omega(x,y) = \cos(3x^{k(k+1)} - 2y^{k(k+2)})\,(x\,dx - y^2 dy)$

$U(x,y) = \alpha \cdot \sin(3x^{k(k+1)} - 2y^{k(k+2)})$

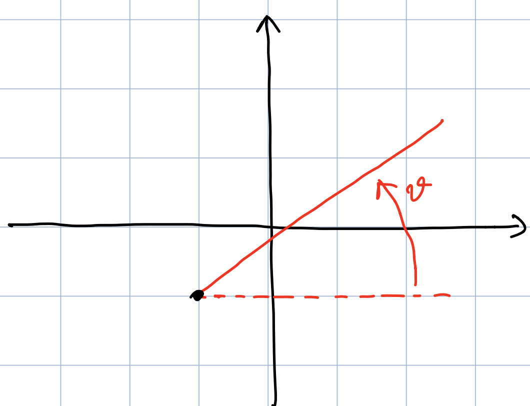
$$+\alpha \cdot \cancel{\cos(\dots)} \cdot 3k(k+1)\, x^{k(k+1)-1} = \cancel{\cos(\dots)} \cdot x$$

$$+\alpha \cdot \cancel{\cos(\dots)} \cdot -2k(k+2) \cdot y^{k(k+2)-1} = \cancel{\cos(\dots)} \cdot (-y^2)$$

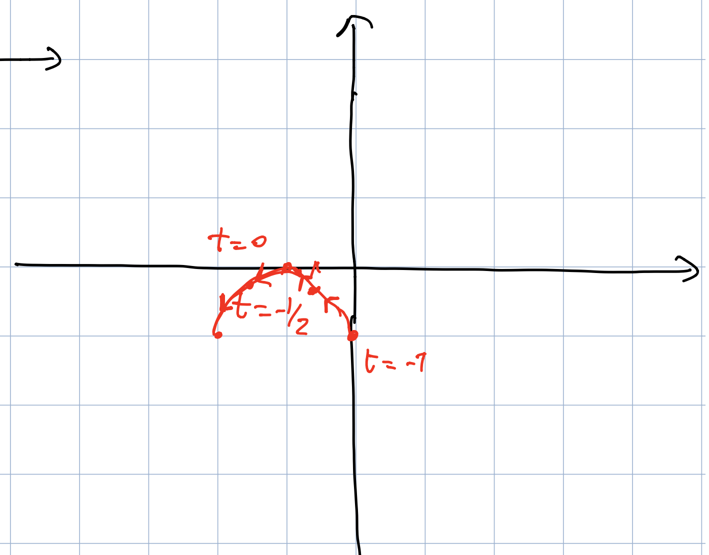
$$\begin{cases} k(k+1)-1 = 1 \\ +\alpha \cdot 3k(k+1) = 1 \\ k(k+2)-1 = 2 \\ -2\alpha\, k(k+2) = -1 \end{cases} \rightsquigarrow \quad \begin{matrix} k = 1 \\ \alpha = 1/6 \end{matrix}$$

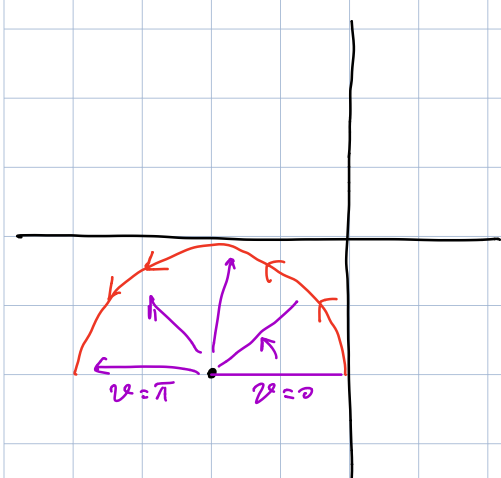
15.2.4 $\displaystyle\int_\alpha \omega \qquad \omega = \frac{(x+1)\,dy - (y+1)\,dx}{(x+1)^2 + (y+1)^2}$

$$\alpha : [-1,1] \to \mathbb{R}^2 \qquad \alpha(t) = -\begin{pmatrix} t+1 \\ t^2 \end{pmatrix}$$



$\omega = d\vartheta$





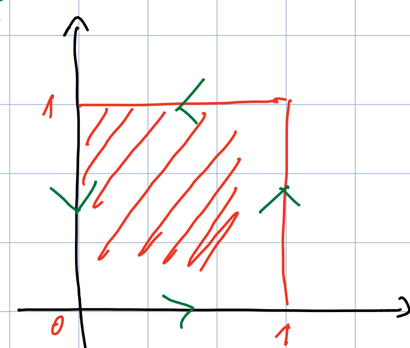
$$\int_\alpha \omega = \pi$$

15.3.1 (b) Calcolare $d\omega$ $\qquad \omega = \frac{y - \ln(x+y)}{x^2 - y^3}\,dx + \boxed{\frac{x^3 - y^2}{xy + \ln(x+y)}}\,dy$

$$d(f\,dx + g\,dy) = \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy$$

$$d\omega = \left(\frac{3x^2\left(xy + \ln(x+y)\right) - (x^3 - y^2)\cdot\left(y + \frac{1}{x+y}\right)}{\left(xy + \ln(x+y)\right)^2} - \frac{\left(1 - \frac{1}{x+y}\right)(x^2 - y^3) - \left(y - \ln(x+y)\right)(-3y^2)}{(x^2 - y^3)^2} \right) dx\,dy$$

$\boxed{15.3.4}$ $\qquad \int_{\partial Q} \left((x^2+y^2)\,dx + (2xy + e^y)\,dy \right) \qquad Q = [0,1] \times [0,1]$



$$= \int_Q (2y - 2y)\,dx\,dy = 0$$

$\boxed{15.3.6}$ $\qquad \int_\alpha \cos(y)\,(dx + dz) - (x+z)\sin(y)\,dy$

$\alpha : [0, \pi] \to \mathbb{R}^3 \qquad \alpha(t) = \left(t, \sin\left(t^2/\pi\right), t(t - \pi)\right)$

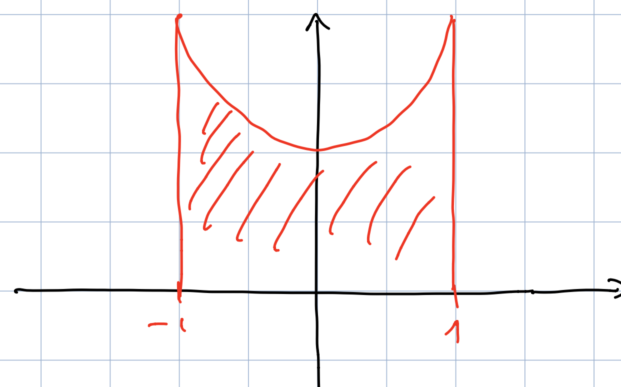
$U(x,y,z) = (x+z)\cdot\cos(y) \qquad \omega = dU$

$$\int_\alpha \omega = U(\alpha(\pi)) - U(\alpha(0)) = U(\pi, 0, 0) - U(0, 0, 0)$$

$$= (\pi+0)\cdot\cos(0) - (0+0)\cdot\cos(0) = \pi$$

$\boxed{15.3.8}$ $\quad \displaystyle\int_{\partial A} x\,dy$, $\quad A = \{(x,y)\in\mathbb{R}^2 : \ |x|\le 1, \ 0\le y\le 1+x^2\}$

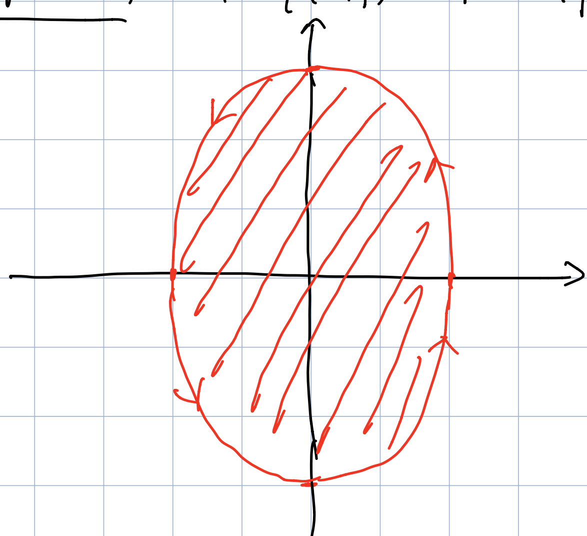
$$\int_{\partial A} x\,dy = \int_A dx\,dy = \text{área}(A)$$



$$= \int_{-1}^{1}\left(\int_0^{1+x^2} dy\right)dx$$

$$= \int_{-1}^{1}(1+x^2)\,dx = 2 + \frac{x^3}{3}\Big|_{-1}^{1} = 2+\frac{2}{3} = \frac{8}{3}$$

$\boxed{15.3.9}$ $\quad A = \{(x,y) : \ 9x^2+4y^2 \le 36\}$, parametrizar ∂A.



$$\alpha(t) = \begin{pmatrix} 2\cos(t) \\ 3\sin(t) \end{pmatrix}$$

$\boxed{15.3.10}$ Verificare che ω è esatta e trovare potenziale

(a) $\omega = -5x^2y\,(3y\,dx + 2x\,dy)$

Def. su $\mathbb{R}^2$: esatta $\iff$ chiusa

$$\omega = -15x^2y^2\,dx - 10x^3y\,dy$$

$$\underbrace{\frac{\partial}{\partial y}\left(-15x^2y^2\right)}_{=\,-30x^2y} \overset{?}{=} \underbrace{\frac{\partial}{\partial x}\left(-10x^3y\right)}_{=\,-30x^2y}$$

$$U(x,y) = -5x^3y^2$$

(c) $\omega(x,y) = \left(2x + y\cdot\sin(xy)\right)dx + \left(x\cdot\sin(xy) - 1\right)dy$

$$\frac{\partial}{\partial y}\left(2x + y\cdot\sin(xy)\right) = \sin(xy) + xy\cdot\cos(xy)$$

$$\frac{\partial}{\partial x}\left(x\cdot\sin(xy) - 1\right) = \sin(xy) + xy\cdot\cos(xy)$$

$$U(x,y) = -\cos(xy) + x^2 - y$$

(e) $\omega(x,y) = y\cdot\sin(x)\,dx + \left(\cos(y) - \cos(x)\right)dy$

$$U(x,y) = \sin(y) - y\cdot\cos(x)$$

21/7/20 (4) $A=\begin{pmatrix} 2i & 3+i \\ -3+i & -i \end{pmatrix}$ trovare base ortog. che diagonalizza

Oss: è antihermitiana: $A^*=-A$

$tr(A) = i = \lambda_1+\lambda_2$

$\det(A) = 2+9+1 = 12 = \lambda_1\cdot\lambda_2$

$\lambda_1 = i\cdot\alpha_1 \quad \lambda_2 = i\cdot\alpha_2$

$\alpha_1+\alpha_2=1$

$\alpha_1\cdot\alpha_2=-12$

$\alpha_1=4 \quad \alpha_2=-3$

$v_1: \begin{pmatrix} 2i & 3+i \\ -3+i & -i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 4i\begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{cases} 2ix+(3+i)y = 4i\cdot x \\ (-3+i)x-iy = 4i\cdot y \end{cases} \qquad \begin{cases} (3+i)y = 2i\cdot x \\ (-3+i)x = 5iy \end{cases}$

$\begin{pmatrix} 3+i \\ 2i \end{pmatrix}$

$v_2: \begin{pmatrix} 2i & 3+i \\ -3+i & -i \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -3i\begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{cases} 2ix+(3+i)y = -3ix \\ \text{—} \end{cases}$

$\begin{pmatrix} 3+i \\ -5i \end{pmatrix}$

Verifico : $\begin{pmatrix}3+i\\2i\end{pmatrix} \perp \begin{pmatrix}3+i\\-5i\end{pmatrix}$

$$(3+i)(3-i)+2i\cdot 5i = 10-10=0 \quad \checkmark$$

(5) $\begin{pmatrix}0 & 3 & -2\\ -3 & 0 & \sqrt{3}\\ 2 & -\sqrt{3} & 0\end{pmatrix}$

antisimmetrica 3×3
$\Rightarrow$ autoval $0, \pm ik$
coniugate tramite ortog. a $\begin{pmatrix}0 & \pm k & 0\\ \mp k & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$

$$p_A(t) = \det\begin{pmatrix}t & -3 & 2\\ 3 & t & -\sqrt{3}\\ -2 & \sqrt{3} & t\end{pmatrix}$$

$$= t^3 - 6\sqrt{3} + 6\sqrt{3} + 4t + 9t + 3t = t(t^2+16) \qquad k=4$$

(6) $(1-t)x^2+4xy+3y^2+2(1+2t)x+2y = t$

$$\begin{pmatrix}1-t & 2 & 1+2t\\ 2 & \boxed{3} & 1\\ 1+2t & 1 & -t\end{pmatrix}$$

$$d_3 = \begin{vmatrix}-1-5t & 0 & 1+4t\\ -1-6t & 0 & 1+3t\\ * & 1 & *\end{vmatrix} = \begin{vmatrix}1+5t & 1+4t\\ 1+6t & 1+3t\end{vmatrix}$$

$$= \begin{matrix}15t^2+8t+1\\ -24t^2-10t-1\end{matrix} = -9t^2-2t = -t(9t+2)$$

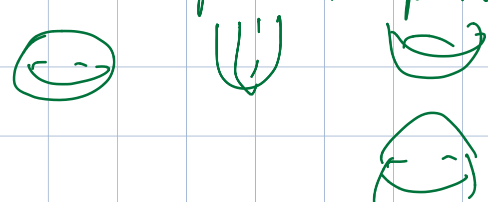
degenere: $t=0$, $t=-2/9$

$d_1>0$ $\quad d_2 = 3-3t-4 = -3t-1$. $\quad d_2=0$ per $t=-1/3$

		$-1/3$		$-2/9$		0	
d_2	$+$	$\bullet$	$-$		$-$		$-$
d_3	$+$		$+$	$\bullet$	$-$	$\bullet$	$+$
	$\emptyset$	parab	ip	deg	ip.	deg.	ip

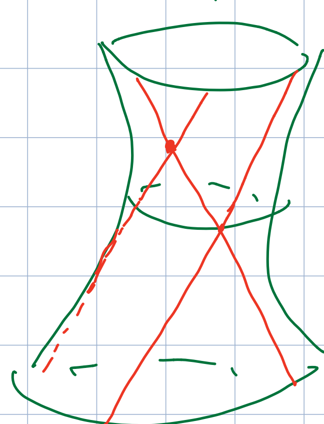
(7) $\#$ rette $\subset Q$ passanti per $P \in Q$

$\emptyset$ ellissoide parab. ell. iperb. ell



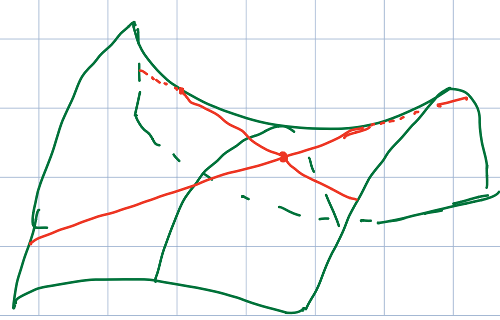
nessuna retta

iperb. iperb.



sempre 2

par. iperb



sempre 2