Geom - Civ - 19/5/21

$\boxed{14.2.1}$ $\alpha : [\frac{\pi}{2}, \pi) \to \mathbb{R}^2$

$$\alpha(t) = \begin{pmatrix} \sin(t) \\ \cos(t) + \log(\tan(t/2)) \end{pmatrix}$$

(1) $L = +\infty$ semplice & (2) τ cambio in p.d'a (3) $\kappa(t)$

$$\alpha'(t) = \begin{pmatrix} \cos(t) \\ -\sin(t) + \frac{1}{\tan(t/2)} \cdot (1 + \tan^2(t/2)) \cdot \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \cos(t) \\ -\sin(t) + \frac{1}{\sin(t)} \end{pmatrix}$$

$Y'(t) > 0 \quad \forall t \Rightarrow Y$ crescente $\Rightarrow \alpha$ semplice

$$\|\alpha'(t)\|^2 = \cos^2(t) + \sin^2(t) - 2 + \frac{1}{\sin^2(t)} = -1 + \frac{1}{\sin^2(t)} = + \frac{\cos^2(t)}{\sin^2(t)}$$

$$\|\alpha'(t)\| = -\cot(t)$$

$$\sigma(t) = \int_{\pi/2}^{t} -\cot(u)\,du = -\ln(\sin(t))$$

$\sigma(t) \to +\infty$ per $t \to \pi \Rightarrow L = +\infty$

$\sigma(t) = s \qquad \tau(s) = \arcsin(e^{-s})$

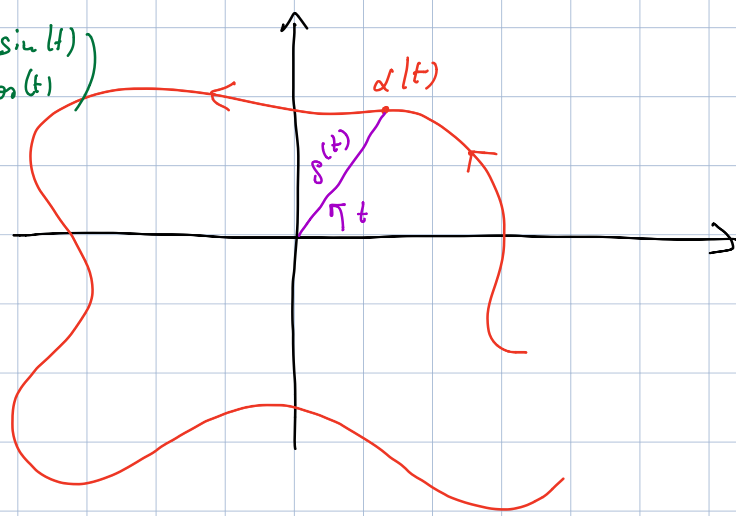
$$\alpha''(t) = \begin{pmatrix} -\sin(t) \\ \dots \end{pmatrix} \qquad \kappa(t) = \frac{\det(\alpha'(t) \;\; \alpha''(t))}{\|\alpha'(t)\|^3} = \dots$$

14.2.3 $\quad \rho : \mathbb{R} \to \mathbb{R} \qquad \alpha(t) = \rho(t) \cdot \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

$$\alpha'(t) = \rho'(t) \cdot \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + \rho(t) \cdot \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix}$$

$$\|\alpha'(t)\|^2 = \rho'(t)^2 + \rho(t)^2$$

$$\sigma(t) = \int_0^t \sqrt{\rho'(u)^2 + \rho(u)^2}\, du$$



$$\alpha''(t) = \rho''(t) \cdot \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + 2\rho'(t) \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + \rho(t) \cdot \begin{pmatrix} -\cos(t) \\ -\sin(t) \end{pmatrix}$$

$\boxed{14.2.4}$ Trovare κ con segno di α in t_0

(b) $\alpha(t) = \begin{pmatrix} 4t - t^5 \\ 4t^2 + t^4 \end{pmatrix} \qquad t_0 = 2$

$$\alpha'(t) = \begin{pmatrix} 4 - 5t^4 \\ 8t + 4t^3 \end{pmatrix} \qquad \alpha''(t) = \begin{pmatrix} -20t^3 \\ 8 + 12t^2 \end{pmatrix}$$

$$\alpha'(2) = \begin{pmatrix} -76 \\ 48 \end{pmatrix} = 4 \begin{pmatrix} -19 \\ 12 \end{pmatrix}$$

$$\alpha''(2) = \begin{pmatrix} -160 \\ 56 \end{pmatrix} = 8 \begin{pmatrix} -20 \\ 7 \end{pmatrix}$$

$$\kappa(2) = \frac{4 \cdot 8 \cdot \det \begin{pmatrix} -19 & -20 \\ 12 & 7 \end{pmatrix}}{4^3 \cdot (19^2 + 12^2)^{3/2}}$$

$\boxed{14.3.2}$ calcolare in ogni pto κ, τ di

$$\alpha(t) = \begin{pmatrix} 1+\cos(t) \\ 1-\sin(t) \\ \cos(2t) \end{pmatrix}$$

$$\alpha'(t) = \begin{pmatrix} -\sin(t) \\ -\cos(t) \\ -2\sin(2t) \end{pmatrix} \qquad \alpha''(t) = \begin{pmatrix} -\cos(t) \\ \sin(t) \\ -4\cos(2t) \end{pmatrix} \qquad \alpha'''(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \\ 8\sin(2t) \end{pmatrix}$$

$\sin(t) = s \qquad \cos(t) = c$

$\sin(2t) = 2sc \qquad \cos(2t) = c^2 - s^2$

$$\kappa = \frac{\| \alpha' \wedge \alpha'' \|}{\| \alpha' \|^3} \qquad \tau = \frac{\langle \alpha' \wedge \alpha'' \mid \alpha''' \rangle}{\| \alpha' \wedge \alpha'' \|}$$

$\boxed{14.3.3}$ Trovare Frenet, κ, τ per α in s_0

(b) $$\alpha(s) = \begin{pmatrix} 1+2s+s^2-s^3 \\ e^{2s} \\ \sin(s) \end{pmatrix}$$

$$\alpha'(s) = \begin{pmatrix} 2+2s-3s^2 \\ 2e^{2s} \\ \cos(s) \end{pmatrix} \qquad \alpha''(s) = \begin{pmatrix} 2-6s \\ 4e^{2s} \\ -\sin(s) \end{pmatrix} \qquad \alpha'''(s) = \begin{pmatrix} -6 \\ 8e^{2s} \\ -\cos(s) \end{pmatrix}$$

$$\alpha'(0) = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \qquad \alpha''(0) = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} \qquad \alpha'''(0) = \begin{pmatrix} -6 \\ 8 \\ -1 \end{pmatrix}$$

Frenet: $$t = \frac{\alpha'(0)}{\|\alpha'(0)\|} = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

n, b : (I) ortonormalizzare $\alpha'(0), \alpha''(0) \rightsquigarrow t, n$

$b = t \wedge n$

(II) $\operatorname{Span}(t, n) = \operatorname{Span}(\alpha'(0), \alpha''(0))$

$\Rightarrow \tilde{b} = \alpha'(0) \wedge \alpha''(0) \qquad b = \tilde{b}/\|\tilde{b}\|$

$n = b \wedge t$

$$\begin{tikzcd} & t \arrow[dr] & \\ b \arrow[ur] & & n \arrow[ll] \end{tikzcd}$$

$$\tilde{b} = \alpha'(0) \wedge \frac{\alpha''(0)}{2} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$$

$$b = \frac{1}{3} \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$$

$$n = b \wedge t = \frac{1}{9} \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \wedge \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} -3 \\ 6 \\ -6 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

$$\kappa = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3} = \frac{2 \cdot 3}{27} = \frac{2}{9}$$

$$\tau = \frac{\langle \alpha' \wedge \alpha'' \mid \alpha''' \rangle}{\|\alpha' \wedge \alpha''\|^2} = \frac{\left\langle 2 \cdot \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \middle| \begin{pmatrix} -6 \\ 8 \\ -1 \end{pmatrix} \right\rangle}{\left\| 2 \cdot \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \right\|^2}$$

$$= \frac{2 \cdot (12 + 8 - 2)}{4 \cdot 9} = \frac{2 \cdot 18}{4 \cdot 9} = 1$$

9/6/20 (9) $\alpha(t) = \begin{pmatrix} t^3 - 5t \\ 3t^2 + 8t \end{pmatrix} \qquad \kappa(1) \qquad \operatorname{sgn}(\kappa(t))$

$$\alpha'(t) = \begin{pmatrix} 3t^2 - 5 \\ 6t + 8 \end{pmatrix} \qquad \alpha''(t) = \begin{pmatrix} 6t \\ 6 \end{pmatrix}$$

$$\alpha'(1) = \begin{pmatrix} -2 \\ 14 \end{pmatrix} \qquad \alpha''(1) = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

$$\kappa(1) = \frac{(2\cdot \det\begin{pmatrix}-1 & 1\\ 7 & 1\end{pmatrix})}{(2\cdot\sqrt{50})^3} = \frac{-96}{20\sqrt{2}} = \dots$$

$$\operatorname{sgn}(\kappa(t)) = \operatorname{sgn}\det(\alpha'(t), \alpha''(t)) = \operatorname{sgn}\det\begin{pmatrix}3t^2-5 & t\\ 6t+8 & 1\end{pmatrix}$$

$$= \operatorname{sgn}(3t^2-5-6t^2-8t) = -\operatorname{sgn}(3t^2+8t+5)$$

$$t_{1,2} = \frac{-4 \pm \sqrt{16-15}}{3} = \frac{-4\pm 1}{3} = \begin{cases} -5/3 \\ -1 \end{cases}$$

neg. per $t < -5/3$, $t > -1$

nulla $t = -5/3$ $t = -1$

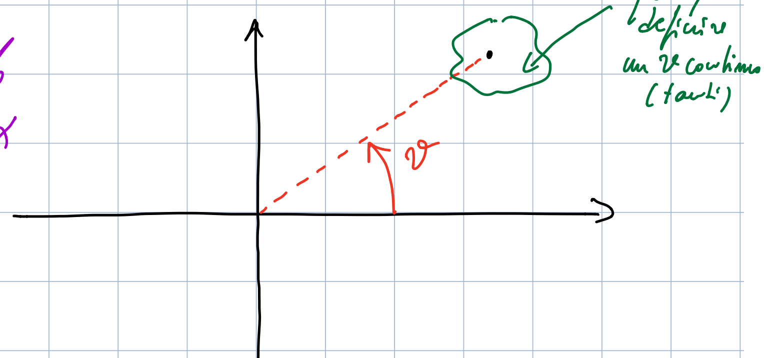
positiva $-5/3 < t < -1$

$\boxed{9/6/20 \quad 10}$

Calcolare $\displaystyle\int_\alpha \underbrace{\frac{-y\,dx + x\,dy}{x^2+y^2}}_{d\vartheta}$

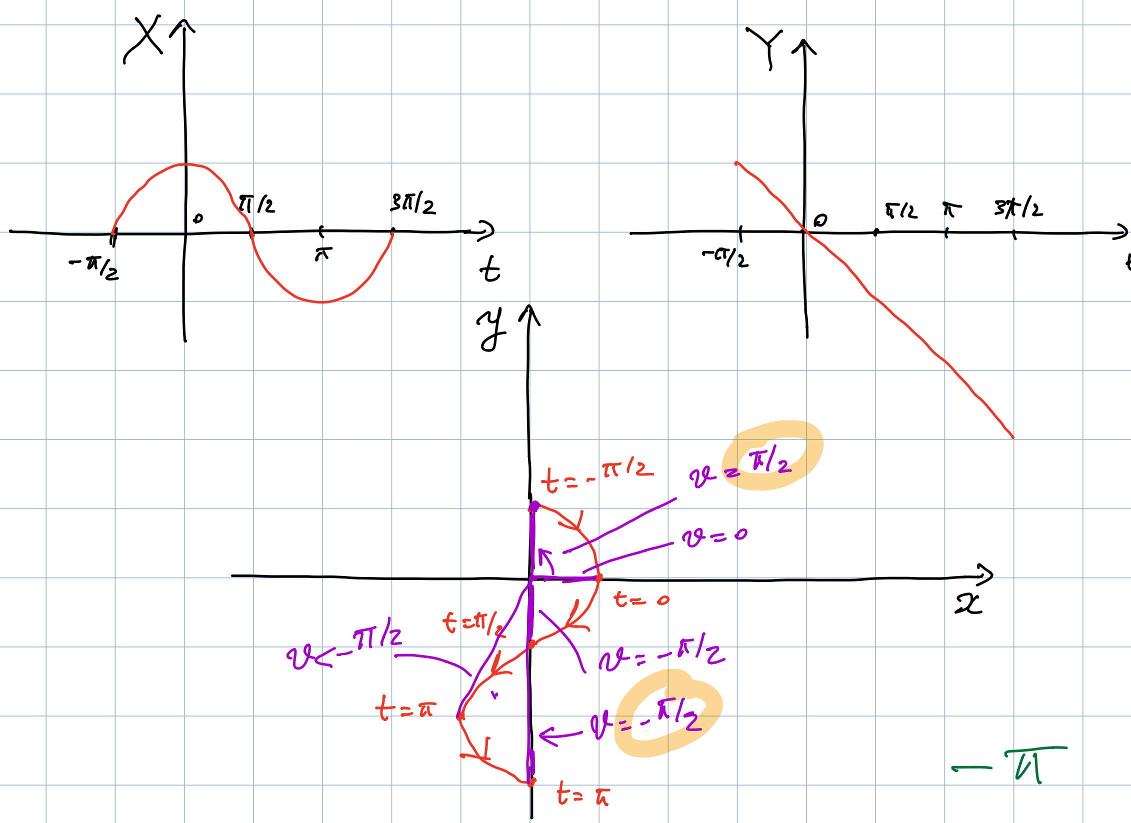
$\alpha : [-\pi/2, \tfrac{3}{2}\pi] \to \mathbb{R}^2 \qquad \alpha(t) = \begin{pmatrix}\cos(t)\\ -t\end{pmatrix}$

$\displaystyle\int_\alpha d\vartheta$ = variazione dell'angolo ϑ lungo la curva α per una ϑ continua lungo α.



Morale: meglio ese $\displaystyle\int_\alpha d\vartheta$ bisogna disegnare α e capire come varia ϑ lungo esse.

$$\alpha : \left[-\pi/2, \tfrac{3}{2}\pi\right] \to \mathbb{R}^2 \qquad \alpha(t) = \begin{pmatrix} \cos(t) \\ -t \end{pmatrix}$$



30/6/20 (9)

$$\alpha(t) = \begin{pmatrix} t \cdot \cos(t) \\ \sin(t) \\ t + t^2 + t^3 \end{pmatrix} \qquad \kappa, \tau \qquad t=0$$

$$\alpha'(t) = \begin{pmatrix} \cos(t) - t\cdot\sin(t) \\ \cos(t) \\ 1+2t+3t^2 \end{pmatrix} \qquad \alpha''(t) = \begin{pmatrix} -2\sin(t) - t\cdot\cos(t) \\ -\sin(t) \\ 2+6t \end{pmatrix} \qquad \alpha'''(t) = \begin{pmatrix} -3\cos(t)+t\ldots \\ -\cos(t) \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$$

$$\alpha' \wedge \alpha'' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

$$K = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3} = \frac{2\sqrt{2}}{3\cdot\sqrt{3}}$$

$$\tau = \frac{\langle \alpha' \wedge \alpha'' | \alpha''' \rangle}{\|\alpha' \wedge \alpha''\|^2} = \frac{-6+2}{4+4} = -\frac{1}{2}$$

(10) $\int_\alpha \omega = 0 \quad \forall \alpha$ chiuse per $k, h = \dots$

ω 1-forma su $\mathbb{R}^2$ (con parametri k, h)

$\int_\alpha \omega = 0 \quad \forall \alpha$ chiuse in $\mathbb{R}^2$

$\iff \omega$ esatta su $\mathbb{R}^2$

$\iff \omega$ chiusa su $\mathbb{R}^2$

perché $\mathbb{R}^2$ è semplicemente connesso

sempre

io: Ω connesso

Lisca: Ω connesso per archi

La vera def. di connesso è un'altra, ma per Ω aperto è equivalente.

$$\omega = \left(k x^2 y^5 - 4 x^3 y^2\right) dx + \left(5 x^3 y^4 + h x^4 y\right) dy$$

I: cerco potenziale

$$U = 1\cdot x^3 y^5 + (-1)\cdot x^4 \cdot y^2$$

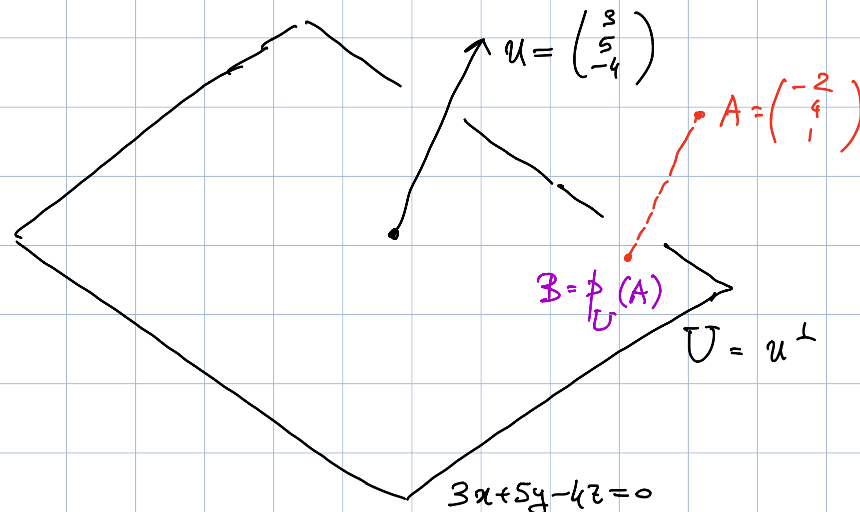
$\Rightarrow k = 3 \qquad h = -2$

II: $$\frac{\partial}{\partial y}\left(k x^2 y^5 - 4x^3 y^2\right) = \frac{\partial}{\partial x}\left(5x^3 y^4 + h x^4 y\right)$$

$$5k x^2 y^4 - 8x^3 y = 15 x^2 y^4 + 4h x^3 y$$

$$k=3 \qquad h=-2$$

21/7/20 ②



$$p_U(A) = A - p_{U^{\perp}}(A) = A - p_{\mathrm{Span}(u)}(A)$$

$$= \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} \middle| \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix} \right\rangle}{\left\| \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix} \right\|^2} \cdot \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} - \frac{-6+20-4}{9+25+16} \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} - \frac{1}{5}\begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -13 \\ 15 \\ 9 \end{pmatrix}$$

③ $\|u\| = 1$, $u_1 + u_2 \in \mathbb{R}$, $u \perp \begin{pmatrix} 1-i \\ 2+i \end{pmatrix}$

I: $u = \begin{pmatrix} a+ib \\ c-ib \end{pmatrix}$ $\quad \begin{cases} a^2+b^2+c^2+b^2 = 1 \\ (a+ib)(1+i)+(c-ib)(2-i) = 0 \end{cases}$

II: trovato u che soddisfa 2ª e 3ª
tutti gli altri sono i suoi multipli reali
Quindi trovo u che soddisfa 2ª e 3ª
e poi risposta $\pm \dfrac{u}{\|u\|}$

Cerco $u = \begin{pmatrix} z \\ 1-z \end{pmatrix}$ $\quad z \in \mathbb{C}$

$$z \cdot (1+i) + (1-z)(2-i)$$

$$z(1+i-2+i) + (2-i) = 0$$

$$z = \frac{2-i}{1-2i} = \frac{(2-i)(1+2i)}{5} = \frac{4+3i}{5}$$

$u = \begin{pmatrix} 4+3i \\ 1-3i \end{pmatrix}$ Risp: $\pm \dfrac{1}{\sqrt{16+9+1+9}} \cdot \begin{pmatrix} 4+3i \\ 1-3i \end{pmatrix}$